Part #1

① (a) Calculate the equation of the tangent line to $y=f(x)$ at $x=8$. $f(x)=x^{1/3}$.

$f(8)=8^{1/3}=2$, so we are finding the tangent line to $y=f(x)$ at $(8,2)$.

$$f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3\sqrt[3]{x^2}}$$

$$f'(8)=\frac{1}{3\sqrt[3]{8^2}}=\frac{1}{3(4)}=\frac{1}{12}$$

$$y-(2)=\frac{1}{12}(x-8)$$

$$\boxed{y=\frac{1}{12}(x-8)+2}$$

(b) what is the linearization of $f(x)$ at $x=8$? Voilà, it's the boxed result from (a).

(c) use your answer from part (b) to estimate $\sqrt[3]{9}$.

$$y(9)=\frac{1}{12}(9-8)+2$$
$$=\frac{1}{12}+2$$
$$\approx \boxed{2.083}$$ → calculator's result is 2.080, so this estimation is not too shabby.

PART #2

① A ball of radius 3 is slightly inflated to radius 3.06.

(a) Estimate the increase in volume using linear approximations.

Volume as a function of radius.

$V(r)=\frac{4}{3}\pi r^3$

$V(3)=\frac{4}{3}\pi(3)^3=36\pi$, so we are finding the tangent line to $V(r)$ at $(3, 36\pi)$.

$V'(r)=\frac{4}{3}(3)\pi r^2=4\pi r^2$

$V'(3)=4\pi(3)^2=36\pi$.

$y-(36\pi)=36\pi(x-3)$

$y=36\pi(x-3)+36\pi$ (our linear approximation of volume as a function of radius)

$y(3.06)=36\pi(3.06-3)+36\pi$

$=36\pi(0.06)+36\pi$

$=\boxed{119.8832 \text{ units}^3}$ → calculator's result is $\boxed{120.0198 \text{ units}^3}$

(b) Surface Area?

$S(r)=4\pi r^2$

$S(3)=4\pi(3)^2=36\pi$; Estimate SA of radius 3.06 with a linear approximation of $S(r)$ at (3,36

$S'(r)=4(2)\pi(r)$

$S'(3)=4(2)\pi(3)=24\pi$.

$y-(36\pi)=24\pi(x-3)$

$y=24\pi(x-3)+36\pi$

$y(3.06)=24\pi(3.06-3)+36\pi$

$=24\pi(.06)+36\pi$

$=\boxed{117.6212 \text{ units}^3 \rightarrow \text{actual is } 117.6665 \text{ units}^3}$