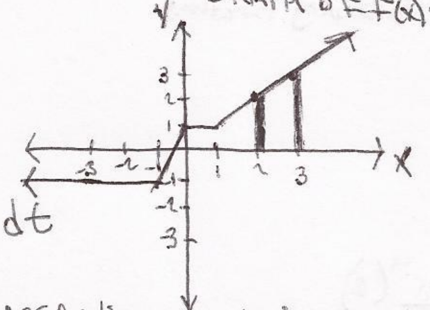
GROUP WORK, May 4

## PART 1

Define $g(x) = \int_1^x f(t)\,dt$

GRAPH OF F(x):




① (a) Shade in the area corresponding to $g(2.1) - g(2.0)$.

(b) Estimate $\dfrac{g(2.1) - g(2.0)}{(0.1)}$.

Consider that $g(2.1) - g(2.0) \approx$ Area of a rectangle beneath the curve from $x = 2.0$ to $2.1$.
Well, Area = bh, and $A = 2(0.1) \approx g(2.1) - g(2.0)$.

So, $\dfrac{g(2.1) - g(2.0)}{(0.1)} \approx \dfrac{(2.0)(0.1)}{(0.1)} = \boxed{2}$.

In other words, $\dfrac{g(2.1) - g(2.0)}{(0.1)} \approx$ height of a rectangle containing the area between $x = 2.0$ to $2.1$.

(c) Guess the value of $\lim_{h \to 0} \dfrac{g(2+h) - g(2)}{h}$.

We've done this before - a good estimate may be found by plugging in values close to $x = 0$.
I'll choose 0.1.

$$\lim_{h \to 0} \frac{g(2+h) - g(2)}{h} \approx \frac{g(2+0.1) - g(2)}{(0.1)} \approx \frac{g(2.1) - g(2)}{(0.1)}.$$ Aha!

We estimated that in part (b) to be $\boxed{2}$.

② (a) Shade in the area corresponding to $g(3.1) - g(3.0)$.

(b) Estimate the following quantity: $\dfrac{g(3.1) - g(3.0)}{(0.1)}$.

Again, this is like the height of the small rectangle representing the area between $X = 3.0$ and $3.1$.
It is about $\boxed{3}$

(c) Guess the value of

$$\lim_{h \to 0} \frac{g(3+h) - g(3)}{h} \approx \frac{g(3+0.1) - g(3)}{(0.1)} \approx \boxed{3}.$$

I followed the same process as in 2 part b.

③ Use your answers from above to estimate

$$\lim_{h \to 0} \frac{g(a+h) - g(a)}{h}.$$

Well, considering that this formula appears to estimate the height of a small rectangle of area squeezed between two values of X and the curve,

$$\lim_{h \to 0} \frac{g(a+h) - g(a)}{h} = f(a),$$ where $f(a)$ is the value of $f$ at $a$,

or, in other words, the height of an infinitesimally small rectangle beneath the $f(a)$.

PART 2 Find the derivatives of the following functions:

① (a) $g(x) = \int_0^x e^{t^2} \cos^{-1}(p)\, dp$

$g'(x) = \frac{d}{dp} \int_0^x e^{t^2} \cos^{-1}(p)\, dp$

$\boxed{= e^{t^2} \cos^{-1}(x)}$

(b) $g(x) = \int_{-1}^x \cos(\tan(t))\, dt$

$g'(x) = \frac{d}{dt} \int_{-1}^x \cos(\tan(t))\, dt$

$= \boxed{\cos(\tan(x))}$

(c) $g(x) = \int_x^{10} r^n\, dr$

$= -\int_{10}^x r^n\, dr$

$g'(x) = -\frac{d}{dr} \int_{10}^x r^n\, dr$

$= \boxed{-x^x}$

(d) $g(x) = \int_x^{x^2} \ln(s^2+1)\, ds$

$g'(x) = \frac{d}{dx} \int_x^{x^2} \ln(s^2+1)\, ds$

$= \frac{d}{dx} \int_x^w \ln(s^2+1)\, ds, \quad w = x^2$

$= \ln(w^2+1)(w)' - \ln(x^2+1), \quad w = x^2, \; w' = 2x$

$= \boxed{\ln(x^4+1)(2x) - \ln(x^2+1)}$