Some Common Inequalities

1. **AM-GM Inequality:** Those letters stand for Arithmetic Mean - Geometric Mean Inequality. It states that for any positive numbers $a_1, a_2, \ldots, a_n$,

   $$\frac{a_1 + \cdots + a_n}{n} \geq (a_1 a_2 \ldots a_n)^{1/n}$$

   with equality attained precisely when $a_1 = a_2 = \cdots = a_n$.

   The name comes from the fact that the expression on the left-hand side is called the arithmetic mean of the numbers $a_1, a_2, \ldots, a_n$, whereas the expression on the right-hand side is the geometric mean.

2. **The Power Mean Inequalities:** The above is actually a specific instance of the power mean inequality. Let $a_1, a_2, \ldots, a_n$ be positive real numbers. Then, if $r \neq 0$, we define the rth power mean of the numbers to be

   $$P_r(a_1, \ldots, a_n) = \left(\frac{a_1^r + a_2^r + \cdots + a_n^r}{n}\right)^{1/r}$$

   If $r = 0$, we define the 0th power mean to be the geometric mean:

   $$P_0(a_1, \ldots, a_n) = (a_1 a_2 \ldots a_n)^{1/n}$$

   Then, the Power Mean inequality states that if $r \leq s$, then the rth power mean is at most the sth power mean, with equality attained if and only if $a_1 = a_2 = \cdots = a_n$. That is, if $r \leq s$,

   $$P_r(a_1, \ldots, a_n) \leq P_s(a_1, \ldots, a_n)$$

   For example, if $r = 1$ and $s = 2$, this says that

   $$\frac{a_1 + a_2 + \cdots + a_n}{n} \leq \left(\frac{a_1^2 + a_2^2 + \cdots + a_n^2}{n}\right)^{1/2}$$

   If $r = 0$ and $s = 1$, we get the AM-GM Inequality!

3. **The Cauchy-Schwarz Inequality** If $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ are real numbers (not necessarily positive!), then

   $$(a_1 b_1 + \cdots + a_n b_n)^2 \leq (a_1^2 + \cdots + a_n^2)(b_1^2 + \cdots + b_n^2)$$