Games and Strategies Questions

1. Make a row of 3 counters, a row of 4 and a row of 5. Two players each take turns to remove any number of counters from a particular row. The player left with the last counter is the loser. Who has the winning strategy?

2. Consider the following two-player game. Each player takes turns placing a penny on the surface of a rectangular table. No penny can touch a penny which is already on the table. The table starts out with no pennies. The last player who makes a legal move wins. Does the first player have a winning strategy?

3. Alice and Bob play a game in which they take turns removing stones from a heap that initially has n stones. The number of stones removed at each turn must be one less than a prime number. The winner is the player who takes the last stone. Alice plays first. Prove that there are infinitely many n such that Bob has a winning strategy.

4. Two players A and B play the following game. A thinks of a polynomial with nonnegative integer coefficients. B must guess the polynomial. B has two shots: she can pick a number and ask A to return the polynomial value there, and then she has another such try. Can B win the game?

5. Consider the following game played with a deck of $2n$ cards numbered from 1 to $2n$. The deck is randomly shuffled and n cards are dealt to each of two players. Beginning with A, the players take turns discarding one of their remaining cards and announcing its number. The game ends as soon as the sum of the numbers on the discarded cards is divisible by $2n + 1$. The last person to discard wins the game. Does either A or B have a winning strategy?

6. There are 2010 boxes labeled $B_1, B_2, \ldots, B_{2010}$, and $2010n$ balls have been distributed among them, for some positive integer n. You may redistribute the balls by a sequence of moves, each of which consists of choosing an i and moving *exactly* i balls from box B_i into any one other box. For which values of n is it possible to reach the distribution with exactly n balls in each box, regardless of the initial distribution of balls?

7. A game starts with four heaps of beans, containing 3,4,5 and 6 beans. The two players move alternately. A move consists of taking **either**

   (a) one bean from a heap, provided at least two beans are left behind in that heap, **or**

   (b) a complete heap of two or three beans.

   The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy.

8. An integer n, unknown to you, has been randomly chosen in the interbal $[1, 2002]$ with uniform probability. Your objective is to select n in an **odd** number of guesses. After each incorrect guess, you are informed whether n is higher or lower, and you must guess an integer on your next turn among the numbers that are still feasibly correct. Show that you have a strategy so that the chance of winning is greater than $2/3$.