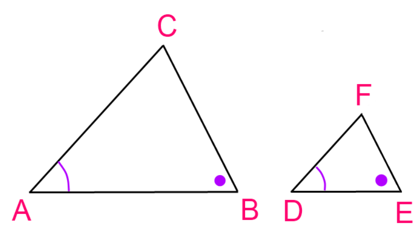
Geometry Handout

1. **General Hints:** Draw a GOOD diagram. Label the diagram carefully. Make sure to find which angles and which lengths are equal; find all the right angles. Give names to angles and lengths if need be, and calculate things in terms of them.

   Also, don't forget things that are common knowledge! The total sum of an angle of a triangle is 180°, the Pythagoras theorem – these can all come in handy.

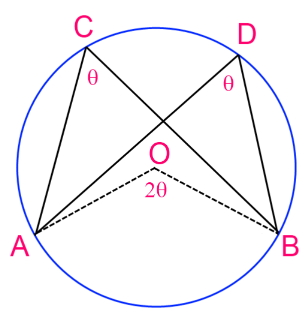
2. **Similar Triangles:** This comes up very often! Say that triangles $\triangle ABC$ and $\triangle DEF$ have the same angles, as in the diagram below: (Note that equal angles are marked with the same symbol; also note that if two pairs of angles are equal, the remaining angle is equal by definition.)



   In the set up above, the triangles have the same shape and hence

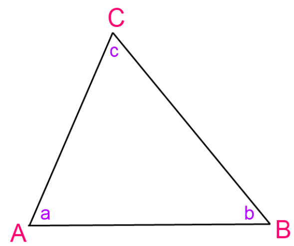
$$\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|} = \frac{|BC|}{|DF|}$$

3. **Inscribed Angles:** An angle inscribed in a circle is half of the central angle that subtends the same arc on the circle: therefore, two angles based on the same arc are the same. This is easiest explained with a picture:



In this picture, the angles $\angle ACB$ and $\angle ADB$ are both based on the arc AB and as such are equal; they are also both half of the angle $\angle AOB$. An easy consequence of this is that an angle based on the diameter of a circle is a right angle.

4. Area Formula and the Law of Sines: Say we have $\triangle ABC$ with the labelled as in the picture below:



Then, we have that

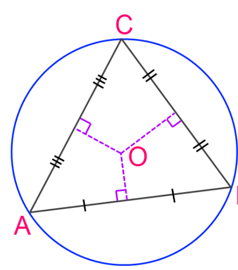
$$\text{Area of } \triangle ABC = |AB||AC|\sin(a) = |AB||BC|\sin(b) = |AC||BC|\sin(c)$$

This also easily yields the Law of Sines:

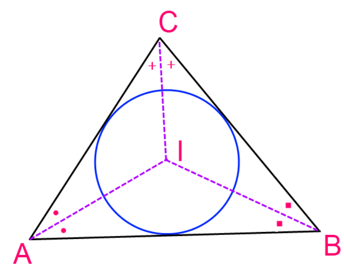
$$\frac{\sin(a)}{|BC|} = \frac{\sin(b)}{|AC|} = \frac{\sin(c)}{|AB|}$$

5. **Triangle Centers:** Triangles have all manner of centers. Here's a list of the common ones:

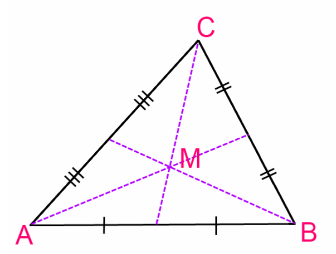
   (a) **The Circumcenter:** This is the center of the circle which goes through each of the vertices of the triangle, called (unsurprisingly) the circumcircle. It is also the intersection of the three perpendicular bisectors of the sides of the triangle. It's often labelled with O, as in the picture below:



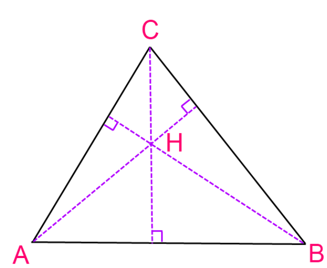
(b) **The Incenter:** This is the center of the circle which is tangent to all the sides of the triangle, called the incircle. It's the intersection of the three bisectors of the angles of the triangle. It is commonly labelled with I:



(c) **The Centroid:** This is the intersection of the three medians of the triangle. A median of a triangle is a line from the vertex to the midpoint of the opposite side, as shown in the picture:



(d) **The Orthocenter:** This is the intersection of the three altitudes of the triangle, often called H. It's shown below:



(e) **Euler line:** Finally, the orthocenter H, circumcenter O and centroid M are collinear: they are on the same line. Furthermore, M always lies between O and H, is twice closer to O than to H; that is,

$$|HM| = 2|MO|$$