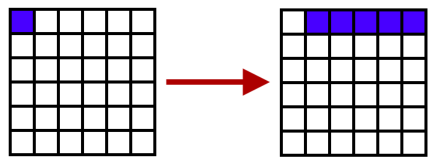
Parity Questions (First Meeting):

1. In a 6×6 chart all but one corner blue square are painted white. You are allowed to repaint any column or any row in the chart (i.e., you can select any row or column and flip the color of all squares within that line). Is it possible to attain an entirely white chart by using only the permitted operations?

   Example operation:

2. John and Pete have three pieces of paper. Each of the boys picks one piece, tears it up, and puts the smaller pieces back. John only tears a piece of paper into 3 smaller pieces while Pete only tears a piece of paper into 5 smaller pieces. After a few minutes can there be exactly 100 pieces of paper?

3. All natural numbers from 1 to 101 are written in a row. Can the signs "+" and "−" be placed between them so that the value of the resulting expression is 0?

4. Of 101 coins, 50 are counterfeit, and they differ from the genuine coins in weight by 1 gram. Peter has a scale in the form of a balance which shows the difference in weight between the objects placed on each pan. He chooses one coin, and wants to find out whether it is counterfeit. Can he do this in one weighing?

5. Suppose a, b and c are integers such that the equation $ax^2 + bx + c = 0$ has a rational solution. Prove that at least one of the integers a, b and c must be even.

6. Can a convex nonagon (a polygon with 9 sides) be cut into parallelograms?

7. Consider a football conference with 13 teams. Is it possible to schedule games so that each team plays exactly 9 games within the conference?

8. (2002, A3) Let $n \geq 2$ be an integer and T_n be the number of nonempty subsets S of $\{1, 2, 3, \ldots, n\}$ with the property that the average of the elements of S is an integer. Prove that $T_n - n$ is always even.
   Note: Ask me if you don't know what a set is!

9. (2008, A2) Alan and Barbara play a game in which they take turns filling entries of an initially empty 2008×2008 array. Alan plays first. At each turn, a player chooses a real number and places it in a vacant entry. The game ends when all the entries are filled. Alan wins if the determinant of the resulting matrix is nonzero; Barbara wins if it is zero. Which player has a winning strategy?
   Note: Ask me if you don't know much about determinants: I can easily summarize all that's required here!