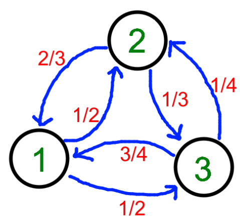
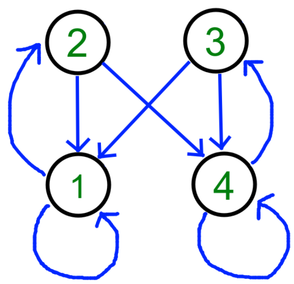
Homework 1

1. For the random walks represented in the following diagrams, calculate the distributions of X_1, X_2, X_3 and X_4.

    (a) [5 pts] Let $X_0 = 1$.

    (b) [5 pts] Let the probability of each arrow be 1/2; let $X_0 = 2$.

2. Draw examples of random walks with the following properties.

    (a) [5 pts] There is no limiting distribution no matter what X_0 is.

    (b) [5 pts] There is a limiting distribution no matter what X_0 is, but it depends on the value of X_0.

    (c) [5 pts] There is no limiting distribution if $X_0 = 1$, but there's a limiting distribution if $X_0 = 2$.

3. Define the following matrices:

$$A = \begin{bmatrix} -2 & 3 & 0 \\ 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 4 & 1 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} -3 & 4 \\ 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Calculate the quantities in the questions below, or explain why it's impossible.

(a) [2 pts] $A - B$.

(b) [2 pts] $AB + C^3$.

(c) [2 pts] BA.

(d) [2 pts] ABC.

(e) [2 pts] $A^2 + B^2$.

4. (BONUS) [5 pts] Write down two sequences of coin flips – one which you generate by writing down a sequence of 20 coin flips "from your head" which is designed to look random, and another for which you use an actual coin to generate 20 flips. Make a note to yourself stating which sequence is truly random. You will get points for this if I can't guess which one is which!