

Homework 2

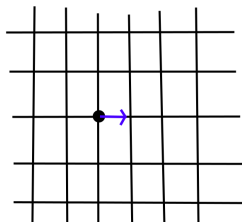
Please show all your work!

1. Define the following matrices:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Calculate the quantities in the questions below, or explain why it's impossible.

- (a) [2 pts] $(AB)^2$
 - (b) [2 pts] A^2B^2
 - (c) [2 pts] $(BA)^2$
2. Toss three fair coins. Calculate the following conditional probabilities:
 - (a) [3 pts] $\mathbb{P}(\text{there are exactly two heads} \mid \text{there is at least one head})$.
 - (b) [3 pts] $\mathbb{P}(\text{there are at least two heads} \mid \text{the first coin lands heads})$.
 3. [3 pts] Roll two six-sided dice. Calculate $\mathbb{P}(\text{the first die is showing a 1} \mid \text{the sum of the dice is 5})$.
 4. Consider the following two walks on an infinite square grid, starting off in the middle going right. For each of the following two random walks, explain whether it is or is not a Markov chain.



- (a) [3 pts] At each intersection, flip two fair coins. If they land HH , go forward; if they land HT , go backward; if they land TH , turn right; and if they land TT turn left.
- (b) [3 pts] At each intersection, flip two fair coins. If they land HH or TT , go forward; if they land HT , go backward; and if they land TH , turn right.