Homework 4

1. Let A be the following matrix:

$$A = \begin{bmatrix} 4 & 1 & -3 \\ -1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

   (a) [3 pts] Check whether the following are right eigenvectors of A. If they are, state the corresponding eigenvalue.

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

   (b) [3 pts] Check whether the following are left eigenvectors of A. If they are, state the corresponding eigenvalue.

$$w_1 = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}, w_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, w_3 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

2. Keeping the notation from class, complete the proof that the periods of two states in an irreducible Markov chain are equal: that is, show that $p(x)$ divides $p(y)$.

   Note: You will also need to introduce some new notation – just keep the relevant notation.

3. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 2/3 & 0 & 1/3 & 0 \end{bmatrix}$$

   (a) Draw the diagram for this chain.

   (b) Calculate the period of each state.

   (c) If you got two states with different periods, why does this not contradict the theorem from class?

4. Consider a Markov chain on two states, 1 and 2, where the probability of staying at 1 at each step is 1/2 and the probability of staying at 2 is 3/4.

   (a) Write down the transition matrix P for the chain.

   (b) Use a high power of P to estimate the stationary distribution of the chain.

   (c) Verify that your guess is indeed the stationary distribution by checking whether it satisfies $\pi P = \pi$.