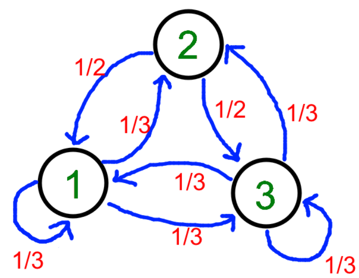
Quiz 1 Solutions

1. For the random walk in the following picture, calculate the distribution of X_2, assuming $X_0 = 3$.



Solution: Clearly, the distribution of X_1 is

$$\mathbb{P}(X_1 = 1) = \mathbb{P}(X_1 = 2) = \mathbb{P}(X_1 = 3) = 1/3$$

Therefore,

$$\begin{aligned}
\mathbb{P}(X_2 = 1) &= \frac{1}{3}\mathbb{P}(1 \to 1) + \frac{1}{3}\mathbb{P}(2 \to 1) + \frac{1}{3}\mathbb{P}(3 \to 1) \\
&= \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{7}{18}} \\
\mathbb{P}(X_2 = 2) &= \frac{1}{3}\mathbb{P}(1 \to 2) + \frac{1}{3}\mathbb{P}(2 \to 2) + \frac{1}{3}\mathbb{P}(3 \to 2) \\
&= \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot 0 + \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{2}{9}} \\
\mathbb{P}(X_2 = 3) &= \frac{1}{3}\mathbb{P}(1 \to 3) + \frac{1}{3}\mathbb{P}(2 \to 3) + \frac{1}{3}\mathbb{P}(3 \to 3) \\
&= \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{3} = \boxed{\frac{7}{18}}
\end{aligned}$$

2. Define

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}.$$

Calculate $A + A^2$.

Solution: By definition,

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix}$$

Therefore,

$$A + A^2 = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} + \begin{bmatrix} 7 & -6 \\ -9 & 22 \end{bmatrix} = \boxed{\begin{bmatrix} 8 & -4 \\ -6 & 18 \end{bmatrix}}$$

3. Draw a diagram for a random walk which has the same limiting distribution regardless of what X_0 is, and such that this limiting distribution isn't equal on all the states.

   Solutions: Let the probability of each arrow in this diagram be $1/2$. Then, states $2, 3$ and 4 have probability $1/3$ in the limiting distribution, while state 1 has probability 0.