Riemann surfaces

T. PERUTZ

The theory of Riemann surfaces—one-dimensional complex manifolds—lies at a crossroads of complex function theory, differential geometry, algebraic geometry, and geometric analysis.

Riemann surfaces also appear in diverse areas of current research interest: in applied mathematics (via conformal mappings); in symplectic topology (via pseudo-holomorphic curves); in number theory (where the ring of holomorphic functions on a Riemann surface is analagous to the ring of integers of number fields); in integrable systems (via Jacobians and other tori associated with Riemann surfaces); in dynamics (e.g. on Teichmüller space); and in low-dimensional topology (e.g. via Heegaard Floer theory).

This is therefore a topic that should be of interest to a broad range of graduate students and advanced undergraduates. The course will be pitched at approximately the same level as a prelim course. It will be based mainly on S. K. Donaldson's text, which emphasizes the connections of Riemann surfaces with other mathematical fields. Students with more experience (those who have studied complex manifolds in general, say) will find that there is much to be learned about the 1-dimensional case. We will tackle some of the main classical results in the subject.

Outline:

- Motivating examples: algebraic functions, analytic continuation, differential equations.
- Riemann surfaces and maps between them; examples; branched coverings and monodromy; the Riemann–Hurwitz formula.
- Calculus on Riemann surfaces (forms, cohomology).
- Elliptic curves and their function theory.
- Riemann–Roch (proved via potential theory).
- Uniformization (sketched).
- Compact Riemann surfaces as algebraic curves; applications of Riemann–Roch.
- Periods and Jacobian tori.

Prerequisites: You will need some familiarity with a number of topics; however, I will not assume deep knowledge of any of them. They are:

- Real analysis: standard undergraduate material; preferably also Hilbert spaces.
- Complex function theory: Cauchy's theorem, residues, power series in a complex variable.
- Algebraic topology: π_1, covering spaces, basics of homology.
- Differential topology: smooth manifolds; differential forms, preferably de Rham cohomology.

Text: S.K. Donaldson, *Riemann surfaces*, Oxford University Press, 2011. You will have access to an ebook edition through the UT library.