

# From Fractal Geometry to Fractured Architecture: The Federation Square of Melbourne

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*Does your hometown have any mathematical tourist attractions such as statues, plaques, graves, the café where the famous conjecture was made, the desk where the famous initials are scratched, birthplaces, houses, or memorials? Have you encountered a mathematical sight on your travels? If so, we invite you to submit to this column a picture, a description of its mathematical significance, and either a map or directions so that others may follow in your tracks.*

Please send all submissions to  
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Melbourne, the second largest city in Australia, has a population of around four million. The capital of the state of Victoria, it lies on the southeast coast of the continent at the mouth of the Yarra River. Interestingly, Melbourne's location was determined by this modest river: in 1835 a city founder proclaimed, 'This is the spot for a village'. Some Melbournians still fondly call the city proper 'the village'. By 1900, the village had exploded to a populous vibrant city, thanks to the Gold Rush and excellent port facilities. Subsequently, an extensive railway network was built to transport people from the city, where they worked, to spreading suburbs.

Thoughtlessly, the builders of the bulk of the railway lines ran them along the Yarra and cut the city in two. By the beginning of the twentieth century, the riverside railway yard of over 50 lines became an eyesore that hampered the development of an otherwise vibrant metropolis. To remove this unsightly railway yard was out of the question; the only rational alternative was to roof it over. This immense engineering prob-

lem generated a second issue: any such development would create new plum real estate at the city's very gateway. What should arise on this deck?

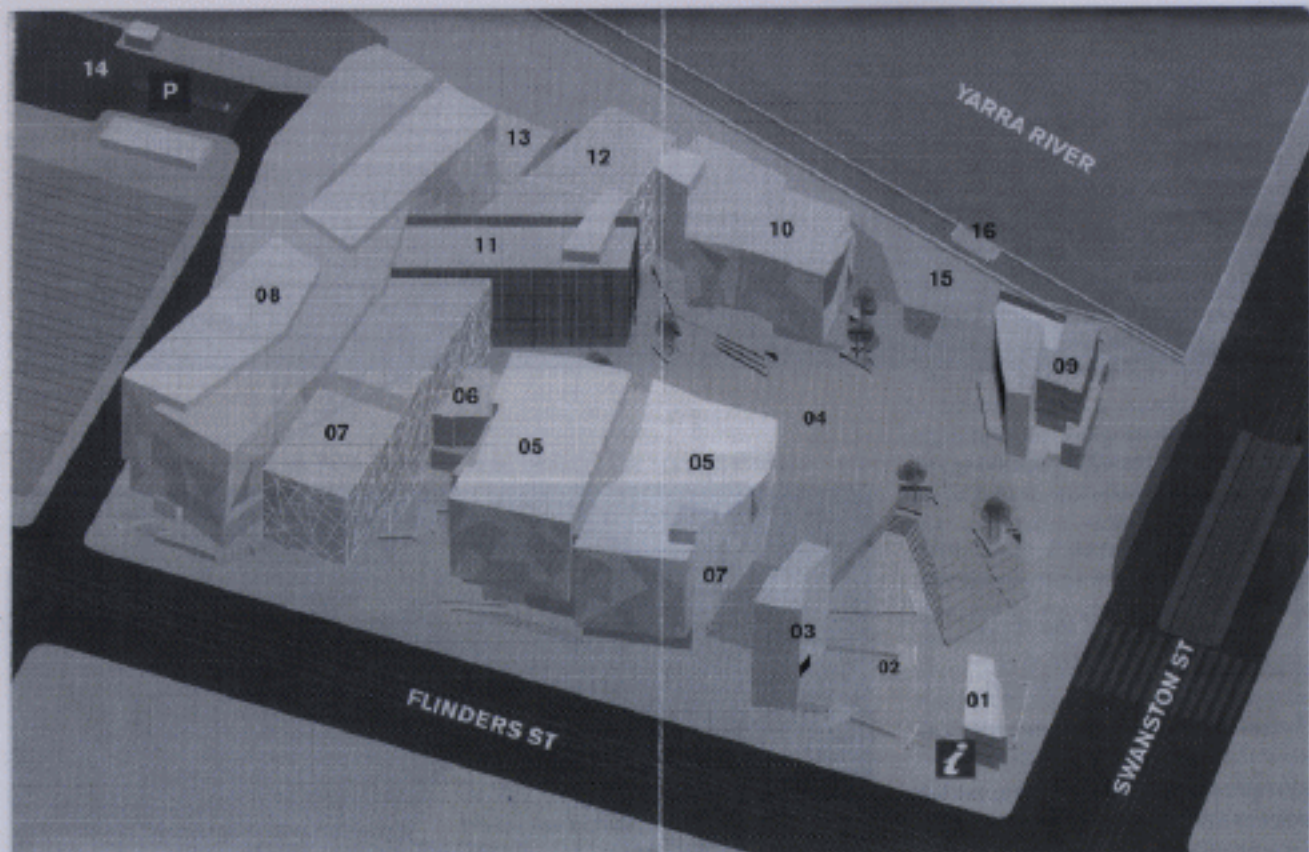
Through the 20th century, countless ideas were proposed for the future of this site. Both private enterprise and government bodies held assorted competitions for the development. Most were just unrealistic dreams. For example, in 1975 some 2300 entries were submitted to a competition held by the Victorian State government. A member of the judging panel described entries of the finalists as 'a megalomania that makes the pyramids look like pimples'.

In the 1990s came the breakthrough. As Australia prepared to celebrate its centennial of Federation in 2001, the Federal Government sought projects appropriate to this celebration. Melbournians recognised in this an opportunity to turn their historical local village development dream into a federation project.

In 1997, with substantial financial support from the Federal Government, the State government of Victoria advertised an international architectural

## The Main Components of the Project

1. A Civic Plaza (or The Square): capable of accommodating up to 15,000 people. The design of the square was the key component in the competition.
2. The Atrium: a covered public thoroughfare which complements the plaza, it can accommodate about 1,000 people.
3. An Art Gallery complex, which is an extension of the world-renowned National Gallery of Victoria (NGV), just over the river from Federation Square. It comprises more than 7000 square metres of exhibition space, and it houses about 22,000 Australian art objects.
4. Australian Centre for the Moving Image: includes several cinemas with the latest technology for preservation, exhibition, and education relating to moving images. This complex complements the Arts Centre across the Yarra, where theatres and a concert hall provide venues for live performances.
5. A variety of eateries, shops, and other service areas, including a riverside reception venue and multi-level car park.
6. All the above elements are built on a deck of 4 hectares (55,000-square metres) roofing the railway yard. The construction of the deck itself is a remarkable engineering achievement.



**Figure 1.** The layout of Federation Square. Key: 01, Information Centre; 02, St Pauls Court; 03, Retail; 04, The Plaza; 05, Cinema Centre; 06, Retail; 07, Atrium (Northern Part); 08, Art Gallery; 09, Hotel; 10, Museums; 11, Retail; 12, Atrium (Southern Part); 13, Function Centre; 14, Car Park; 15, River Terrace; 16, Wharf.

design competition for a centre of cultural activities. From 177 entries world wide, Lab Architectural Studio, based in London, was chosen unanimously by an international judging panel. Subsequently Lab, joined by Bates Smart Architects of Melbourne, was granted the job to develop the Federation Square complex. The principal architects were Peter Davidson and Donald L. Bates.

Before looking at the mathematics used in the project, it is helpful to read some selected comments from contemporary media which reflect the controversies and recognise the novelties in the design (Fig. 1). Most of the commenting authors are professionals in architecture or in art.

Federation Square is as different and peculiar as if it came from another culture altogether [7]

Contrary to the claims for its formal novelty and 'beauty', the design of this complex adheres to the conventions of humanism. [5]

... [It] is the biggest example yet of a paradigm in architecture. [6]

The façade is covered with the wonderful fractal patterning that is so arresting to the passersby. [4]

It is a fusion of mathematics, art and architecture [4]

It is a topological extension of the city. [5]

### Geometry of the Façade— Art of Tilings

The façade of the three main buildings is a multi-faceted, multi-coloured composition, a trilogy in tiling design (Fig. 2). Visually it is comparable to some cubist paintings where the objects appear fragmented, faceted, and broken. So it is quite a surprise to learn that the geometric seed of this complicated-ap-



**Figure 2.**

peating structure is a single triangle, namely a right-angled triangle with proportion of the perpendicular sides 1:2. (It is to be noted that the ratio of the sides of the largest face of a standard brick is also 1:2)

To see how the tiling was developed, let us play with a simple jigsaw puzzle. From a piece of cardboard, cut five identical right-angled triangles whose perpendicular sides measure, say, one centimetre and two centimetres. Assemble the five triangles in such a way as to obtain a larger version of the initial right-angled prototriangle. (This can be obtained by applying: rotations through one of the angles of the prototriangle, vertical reflection, and

*The façade of the three main buildings is a trilogy in tiling design.*

appropriate translation, so that no two adjacent triangles are in the same alignment.) Call this new triangle a panel (Fig. 3). Next, assemble five panels the same way. Again we obtain a right-angled triangle similar to the prototriangle. Call this new triangle a megapanel. Subdivide each panel of the megapanel into prototriangles and observe that in the megapanel, the vertices of eight prototriangles meet at one point so that their sides are in a star-like, pinwheel formation (Fig. 3). The architects used such megapanel as cladding units to assemble the façade, where the pinwheel grid is the characteristic feature. The measurements of the actual building prototile are 0.6 metre, 1.2 metre, and  $\sqrt{1.8}$  metre.

Mathematically we can continue this construction process to obtain an incremental sequence system of self-similar triangles. Radin [8], who designed the pinwheel tiling, also proved that the procedure provides an aperiodic tiling of the euclidean plane. Notice that periodic tiling, obviously possible for two panels, or two of any element of the sequence, can abut along the diagonals to form a rectangular tile.

Notice also, that we can reverse the above procedure: instead of assembling, sub-divide a right-angled triangle of ratio 1:2 into five congruent trian-

gles similar to the initial triangle. Keep on sub-dividing the new smaller triangles or just some of them, and you obtain a decreasing sequence of fractal configurations. For further interesting problems regarding Radin tiling, con-

sult Radin [8, 9] and the references therein.

We perceive that the façade is a finite part of an infinite iterative procedure. The pinwheel tiling starts and ends within the boundaries of the

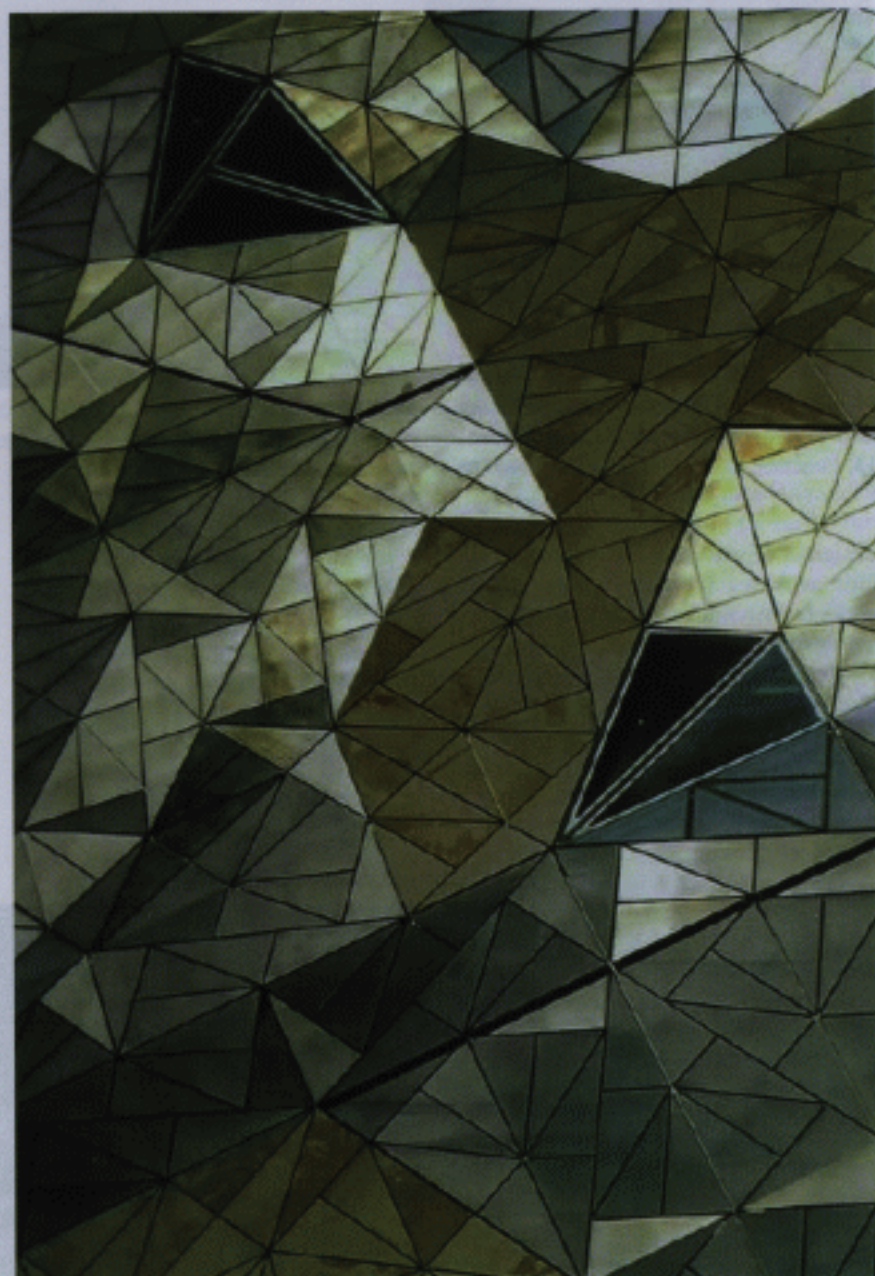


Figure 3.

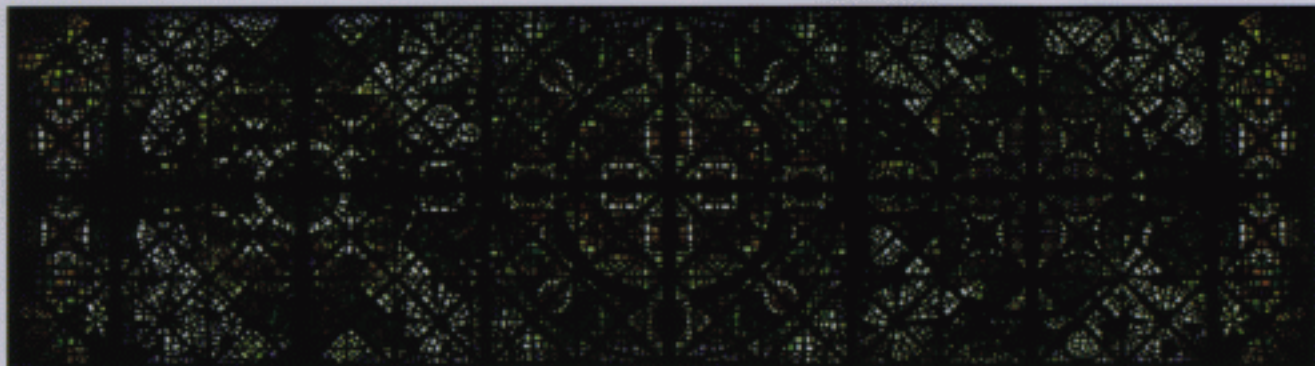


Figure 4.

façade. This project shows how mathematically motivated designs in architecture are restricted and determined by many factors such as construction, function, and material.

Three materials were used for the tiling: glass, sandstone, and zinc. The glass comes in opaque and clear, the sandstone is rough or polished, and the zinc is perforated or solid. The façade contains approximately 22,000 protiles, about 2000 of them of glass, 8000 of sandstone, and 12,000 of zinc. However, there is a fourth material which contributes to the total picture. Through the glass panels, and elsewhere, we can see sections of the corrugated steel structure (Fig. 2) on which the panels are mounted—the skeleton of the façade. The geometry of the behind-the-scenes skeleton gives the facade its 'angled tangled' surface. The architects suc-

*Radin proved that the procedure provides an aperiodic tiling of the euclidean plane.*

ceeded in selecting the materials and their colours to refer to the façades of the neighbouring buildings, despite its unorthodox fractured surface deviating from the usual façades.

Interestingly, even the basic geometric theme, the triangle itself, is featured in many places nearby. I will just mention two examples. One is the landmark Roy Grounds designed 162 metre-high spire of the Victorian Arts Centre (Fig. 2) almost opposite Federation Square. The entire structure is made up of a triangular mesh of sculp-

tural beauty. The second example is the stunning stained glass ceiling of the Great Hall (Fig. 4) in the National Gallery of Victoria, designed and executed by Leonard French. It measures  $51 \times 15$  metres and is constructed of approximately 10,000 pieces of multi-coloured hand-cut glass, embedded in a triangular grid. The twelve supporting columns are topped like the spikes of inverted umbrellas, a three-dimensional pin-wheel formation. (You can best enjoy this wonderful ceiling by lying supine in the finely carpeted hall, relaxing with all the other tourists doing the same! The effect is to make you feel that you are under branches of trees with budding coloured crowns.) In both projects the shape and transla-

tion are very different from that of the façade and, in fact, from each other.

### The Civic Plaza—The Square

The Civic Plaza known as The Square. It is not a 'square' at all but rather an irregularly shaped, fractured square of area 7500 square metres containing several interconnected bays, each of which is designed as a mini-square suitable for intimate gatherings, such as family parties and open-air video showings. This is a fine example of the application of fractal self-similarity in architecture. It is apparently novel to design a plaza in this way. The ground of the plaza has a sloping topography, demanded by the structural need to roof the railway yard. However, the archi-

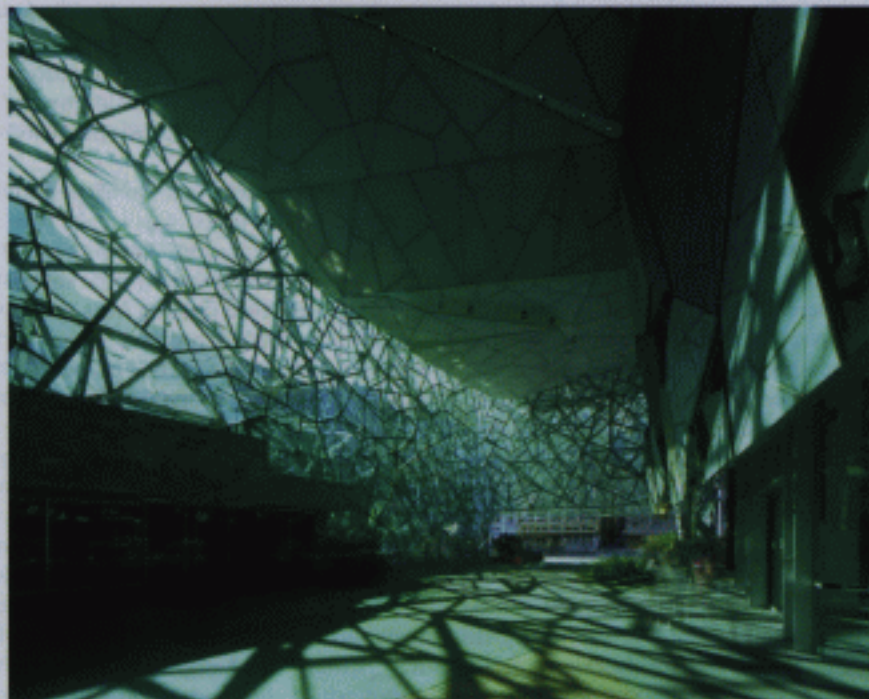


Figure 5

...the same geometric theme of expanding spirals is provided

tect took advantage of this slope by transforming its surface to design a spectacular open amphitheatre. The ground inclines from street level by steps and slopes to a height of about 6 metres at the façade, allowing a panorama towards the river.

The paving of the ground is a remarkable work of art designed by Paul Carter [2]. It is surfaced with sandstone cobblestones laid in seemingly sinusoidal patterns of varying amplitudes or periods. There is a striking contrast between the undulating cobbled ground surface and the all-linear triangulated façade, a backdrop to the plaza. The many-coloured crystal-like cobbles emit a variety of sparkling colours, which change depending on how the sunlight strikes them. There is a delightful interplay between this colour spectrum and the multicoloured façade in the background.

These two elements of the plaza—the ground surface and the façade behind—make a coherent unit. If the artist Christo were to wrap the façade, the ground would definitely appear odd and different. Arguably, Lab Architectural Studio may have won the competition because of this imaginative design concept. Some critics noted that the façade and the interiors of the buildings are not related to each other. It seems these people failed to notice that the façade was designed to relate to the surface of the plaza.

### The Atrium—Tourist in Wonderland

The atrium is the spine of the complex. It consists of three areas. The northern part (Fig. 5) is an entry from Flinders Street leading to the art gallery and cinema centre. In the middle section is a covered amphitheatre with seating capacity of 450, complementing the open amphitheatre of the square. The southern section slopes towards the river, accessed by an impressive stairway. The atrium cuts through the entire complex with a north-south axis providing a covered footbridge over the railway yards, linking the commercial quarters of the city with the Yarra River. It complements the open-air Princes Bridge. Probably necessity will eventually demand a covered footbridge over the river directly from the atrium.

In addition to the atrium's important functional facilities, it is the geometry of the steel network of the surrounding walls that is of particular interest for us. The network is a three-dimensional generalisation of the façade's pinwheel grid. With your megapanel

*The design is a unique avenue of abstract sculptures composed of geometric themes.*

cardboard model, you can visualise this generalisation. Cut out a number of strips of elastic webbing and place them on the spokes of a pinwheel on your model. Fasten the ends of the elastic to the cardboard and then pull the centre ends up from the cardboard plane to the space in arbitrary directions. You will obtain skeletons of conical bodies. In the atrium's structure, the elastic strips are replaced by 200-mm-square hollow sections of corrugated steel tube segments. The tube segments are joined by sophisticated star-like steel connectors. The segments emanate from the connectors in up to six different directions, providing the three-dimensional version of the pinwheel. The intricate connectors made it possible to design a network of skeletons of a variety of three-dimensional configurations, such as tetrahedrons and prisms. The design is a unique avenue of abstract sculptures composed of geometric themes.

The steel structure is enveloped inside and out by tinted glass walls with sandblasted decoration. The glass consists of polygonal panes of nine different shapes that are derived from the façade's right-angled prototriangle. The actual measurement of the perpendicular sites of the 'prototriangles' is 1 metre and 2 metres. Both the steel structure that supports the glass walls inside and the frames which connect the panes contribute to the sculptural effect of the enclosed main structure. Notable is the angled ceiling, made of acoustically effective translucent plastic material, which carries out the polygonal pattern of the glass walls. The geometry of the compound steel net-

work is especially 'transparent' through the huge glass walls surrounding the amphitheatre. It appears as if Escher-type four-dimensional fantasy configurations come alive in our 'real' 3-space.

For a mathematical tourist, an extra night time visit will be rewarding. This is not only to enjoy the intricate theatrical lighting illuminating the atrium with an 'Alice in Wonderland' spell, but also to notice on the ground and elsewhere, pinwheel formations in the shadows projected from the 3-space skeletons branching out of the connectors (Fig. 4). As a bonus, the ever-changing lighting of the Cultural Centre spire is especially dramatic.

### ACKNOWLEDGEMENTS

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