M346 (55820), Homework #1

Due: 12:00pm, Wednesday, Jan. 25

Instructions: Questions are from the book "Applied Linear Algebra, 2nd ed." by Sadun. Please show all your work, not only your final answer, to receive credit. Keep answers organized in the same order the problems have been assigned.

Introduction (1.1)

p. 7, #1, 3, 5, 7

Vector spaces (1.2)

p. 13-14, #2, 4, 5, 6, 8, 9, 10, 12, 13, 14, 19, 20 (*typo*: should be $c \otimes x = cx + c - 1$), 21

In addition:

A) Is the set of all polynomials of degree *exactly* n a subspace of $\mathbb{R}_n[t]$?

B) Is the set of all symmetric $n \times n$ matrices (i.e., satisfying $A^T = A$) a subspace of M_{nn}?

C) Let X and Y be subspaces of V. Show that $X \cap Y$ is a subspace of V.

D) For $X, Y \subset V$ define the *internal direct sum*

$$X + Y = \{\boldsymbol{v} \in V : \boldsymbol{v} = \boldsymbol{x} + \boldsymbol{y} \text{ with } x \in X \text{ and } y \in Y\}.$$

Show that if X and Y are subspaces of V then $X + Y$ is also a subspace.

Matrix operations (Appendix A)

Let $A = \begin{pmatrix} 1 & 1 & 2 & 2 & 1 \\ 2 & 2 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 & 2 \\ 1 & 1 & -1 & -1 & 0 \end{pmatrix}$.

A) Find the reduced row-echelon form of A. What is the rank of A? Describe the column space of A.

B) Find the set of all solutions to $A\boldsymbol{x} = 0$. Does $A\boldsymbol{x} = \boldsymbol{b}$ have a solution if $\boldsymbol{b} = (4, 3, 2, 1)^T$, and if so, what is its most general form?