M346 (55820), Sample Final Exam Questions

Below are some sample final exam questions that cover only the material taught since the last midterm (for material taught prior to this, rework your midterms and sample midterms). Please note that the intent of these is to help prepare for the exam, and that actual exam questions will not merely be modifications of these problems. Consult your HW, lecture notes, and books for additional sources of material to review.

1.

i. For which $z \in \mathbb{C}$ is the sequence $\boldsymbol{v} = (a_1, a_2, a_3, ...)$, $a_n = z^n$, in $l_2(\mathbb{C})$? Why?

ii. For which $p \geq 0$ is the sequence $\boldsymbol{v} = (a_1, a_2, a_3, ...)$, $a_n = (2 + n^p)^{-1}$, in $l_2(\mathbb{C})$? Why?

2. Compute the Fourier sine series, standard (real) Fourier series, and standard (complex) Fourier series of the function $f(x) = \sin(\pi x)$ on the interval $[0, 1]$. [Hint: Use the trigonometric identity $2\sin(u)\cos(v) = \sin(u+v) + \sin(u-v)$, if needed.]

3. Using Fourier sine series, find the solution $u(x, t)$ to the time-dependent Schrodinger equation for a free particle in a 1-dimensional box:

$$\begin{cases} i\partial_t u = -\partial_{xx} u \\ u(0,t) = 0, u(a,t) = 0 \ , \qquad x \in [0, a],\ t \geq 0. \\ u(x, 0) \text{ given} \end{cases}$$

(Here, $i = \sqrt{-1}$ is the imaginary constant.) That is, find the Fourier coefficients of the solution in terms of the Fourier coefficients of the initial data $u(x,\ 0)$. Are the modes of the system stable, neutrally stable, or unstable? How does the solution behave and how does this differ from the heat equation studied earlier?