M346 (56615), Homework #10

Due: 3:30pm, Tuesday, Apr. 16

Instructions: Questions are from the book "Applied Linear Algebra, 2nd ed." by Sadun. Please show all your work, not only your final answer, to receive credit. Keep answers organized in the same order the problems have been assigned.

Gram-Schmidt orthogonalization (6.5)

p. 166-167, #5, 8, 11

Projections (6.6)

p. 169, #1

Least squares (6.7)

p. 174-175, #3, 4

Adjoints

A) On $\mathbb{C}^3$ with the standard inner product, let $A\boldsymbol{x} = (5x_2 - i\,x_3,\, (2+3i)x_1 - 4x_2,\, i\,x_1 + x_3)^T$ where $\boldsymbol{x} = (x_1, x_2, x_3)^T$. Compute the adjoint A^* and determine $A^*\boldsymbol{x}$.

B) Let V be the space of smooth real-valued functions on $\mathbb{R}$ which vanish at infinity, equipped with inner product $\langle f\,|\,g\rangle = \int_{-\infty}^{\infty} f(x)g(x)e^{-x^2/2}dx$. Find the adjoint L_1^* of the linear operator $L_1 = d/dx$ and use this to find the adjoint L_2^* of $L_2 = d^2/dx^2$.

In addition:

A) Find the best fit of the circle $x^2 + y^2 + cx + dy + e = 0$ to the set of points $\{x_i, y_i\}_{i=1}^5 = \{(1,1), (2,0), (3,1), (3,2), (2,2)\}$. Then plot this circle and the data points in the plane for a visual comparison.