M346 (56615), Homework #5

Due: 3:30pm, Tuesday, Feb. 26

Instructions: Questions are from the book "Applied Linear Algebra, 2nd ed." by Sadun. Please show all your work, not only your final answer, to receive credit. Keep answers organized in the same order the problems have been assigned.

Complex eigenvalues and eigenvectors (4.4)

p. 70-71, #2, 6, 8, 9, 12

Diagonalizability (4.5)

p. 76, #4, 5, 6, 9

Jordan canonical form (4.9)

p. 93, #1

[Hint: For #1, if $v \in \tilde{E}_\lambda$ show that $w := (L - \lambda I)v \in \tilde{E}_\lambda$. Now write $Lv = (L - \lambda I)v + \lambda v$.]

In addition:

A) Find the eigenvalues and corresponding eigenspaces of $A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -2 & 1 \end{pmatrix}$. How does the linear transformation $A: \mathbb{R}^3 \to \mathbb{R}^3$ act on the points $a_1 = (1, 0, 0)^T$, $a_2 = (0, 1, 0)^T$, and $a_3 = (0, 0, 1)^T$ (i.e., compute Aa_1, Aa_2, and Aa_3)? Draw the action of A on a cube centered at the origin in order to relate your answer with the geometry of the transformation.

B) Let $A = \begin{pmatrix} -2 & 2 & -1 \\ 0 & 0 & 1 \\ 2 & -2 & 2 \end{pmatrix}$. Write $A = P\tilde{D}P^{-1}$, where $\tilde{D}$ is the Jordan canonical form of the matrix and P is the matrix of power vectors.

C) Suppose a 8×8 matrix has eigenvalues $\lambda = 2$ (with algebraic multiplicity 4 and geometric multiplicity 2), $\lambda = -4$ (with algebraic multiplicity 2 and geometric multiplicity 2), and $\lambda = 1$ (with algebraic multiplicity 2 and geometric multiplicity 1). Is this enough information to write the matrix in Jordan canonical form, and if so, what is it?

Survey (+2 bonus pts.)

Please provide your feedback to the questions on the next page.

M346 (56615), Survey #1

Due: 3:30pm, Tuesday, Feb. 26

Please provide your feedback using the following survey. Be as specific as possible in your comments.

1. How useful have you found the textbook and the lecture notes posted online? Have you had to use other resources? Do you have any feedback regarding assigned homework?

2. Give your comments regarding the quality and relevance of lectures. Are there any particular changes you would like to see made?

3. What is your main purpose in taking this course? If there was one thing you hoped to get out of it, what would it be?

4. Out of the following applied topics, circle the top **three** that are most interesting/relevant to you:

   - Markov chains/network analysis

   - Least squares/data mining/statistics

   - Ordinary differential equations (ODE)

   - Partial differential equations (PDE)

   - Fourier series

   - Other (please specify):