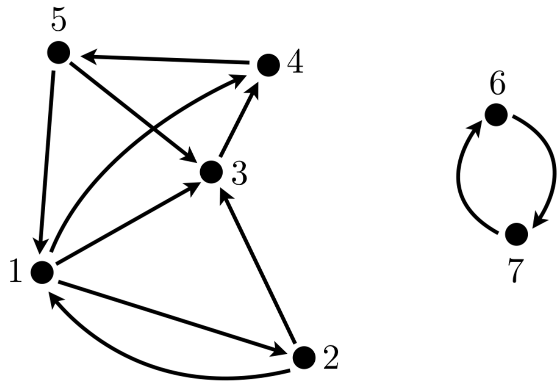
M346 (92153), Homework #8

Due: 10:00am, Thursday, Jun. 21

**Applications to network science**

Consider the following directed network $\mathcal{N}$:




i. Determine the adjacency matrix $A$ of the network $\mathcal{N}$.

ii. What are the in-degrees $d_i^{\text{in}}$ and out-degrees $d_i^{\text{out}}$ of each node $i \in \mathcal{N}$?

iii. Find the number of paths of length 5 from node 1 to node 3 using $A$.

iv. A *cycle* of length $r \geq 1$ is any path of length $r$ that begins and ends at the same node (in particular, we count separately paths consisting of the same nodes in the same order but with different starting points). Compute the total number of cycles of length 6 in the network. Give an asymptotic expression of the form $C(r) \sim a^r$ for the total number of cycles $C(r)$ of length $r$ as $r \to \infty$.

v. Based solely on the visual representation of the network, create your own ranking of the relative importance of each node in decreasing order (e.g., $R = \{2, 6, 3, 5, 1, 4, 7\}$, where node 2 is most important and 7 is least important). Briefly explain your ranking.

vi. Suppose we perform a random walk on the network with transition probabilities from node $i$ given by $A$ normalized by the out-degree $d_i^{\text{out}}$. Write the transition matrix $T$.

vii. Is $T$ irreducible, aperiodic, or both (i.e., regular)? Determine all stationary distributions of the Markov chain with transition matrix $T$.

viii. Determine the PageRank of each node in the network with a damping factor $\alpha = 0.85$ by directly computing the unique eigenvector (which is a probability vector) corresponding to eigenvalue 1. How closely does the relative importance of each node implied by PageRank compare with your qualitative ranking from before?

ix. Instead, use the power method to estimate PageRank with damping factor $\alpha = 0.85$. Find a upper bound by hand for the magnitude of the second largest eigenvalue, and compare the implied rate of convergence to the number of iterations needed for the power method to yield an answer reasonably close to the exact solution.

[Hint: Use a calculator, *MATLAB*, or online program such as *Wolfram Alpha* to perform some of these computations.]