M362K (56310), Homework #1

Due: 12:30pm, Thursday, Jan. 27

Instructions: Please show all your work, not only your final answer, in order to receive credit. Please keep answers organized in the same order the problems have been assigned.

Equally likely outcomes (1.1)

Note: In each of the following problems, create an outcome space and indicate how the event of interest can be represented as a subset of your outcome space.

1. Pitman, p. 09, #2

2. Pitman, p. 10, #5

3. Pitman, p. 10, #7

4. Pitman, p. 10, #8

Probability spaces and distributions (1.3)

5. Pitman, p. 30, #2

6. Pitman, p. 30, #3

7. Pitman, p. 31, #6

8. Pitman, p. 31, #8

9. Pitman, p. 31, #10

10. Pitman, p. 32, #14

Interpretations of probability (1.2)

11. Suppose I ask someone what she believed the chances were of (a) rain today (b) rain tomorrow (c) rain both today and tomorrow (d) rain either today or tomorrow. After some thought she gives 30%, 40%, 20%, and 60%, respectively, as answers. Are these subjective probabilities consistent with the rules (axioms) of probability? Explain why or why not.

Calculus review

12. Evaluate the following (possibly infinite) sums:

$$\text{a)}\ 1+2+\ldots+n, \qquad \text{b)}\ \sum_{k=2}^{\infty} \frac{4}{3^{k+1}}, \qquad \text{c)}\ \sum_{k=1}^{\infty} \frac{e^k}{k^2}$$

13. Compute the following (possibly infinite) integrals:

$$\text{a)}\ \int_1^{\infty} \frac{1}{x(\ln(x))^2}\,dx, \qquad \text{b)}\ \int_0^{\infty} y e^{-y} dy$$

14. Evaluate the double integral

$$\int\int_D e^{-x^2} dx\,dy$$

where D is the triangular region in the xy-plane with vertices $(0,0)$, (1,1), and (1,0).

Puzzle of the week (optional!)

The Monty Hall problem was presented in lecture as follows:

Suppose a prize is randomly placed behind one of the three doors, and I always start by picking door #1. The host then reveals one of the two remaining doors that doesn't have the prize behind it. We showed (using equally likely outcomes) that the probability of winning the prize was 2/3 if we switched and 1/3 if we stayed.

Now suppose that instead always picking door #1 to start with, we begin by choosing one of the three doors at random. (That is, both the placement of the prize and my initial choice of doors are chosen at random.) Do the probabilities of winning if we switch/stay remain the same? Show this explicitly using an appropriate probability space.