M362K (56310), Homework #13

Due: 12:30pm, Thursday, May 05

Instructions: Please show all your work, not only your final answer, in order to receive credit. Please keep answers organized in the same order the problems have been assigned.

Conditional expectation: discrete case (6.2)

1. Pitman, p. 406, #1
2. Pitman, p. 407, #4
3. Pitman, p. 407, #5
4. Pitman, p. 407, #6
5. Pitman, p. 408, #11
6. Pitman, p. 466, #2

Conditioning for continuous r.v.'s (6.3)

7. Pitman, p. 426, #2
8. Pitman, p. 426, #3
9. Pitman, p. 426, #5
10. Pitman, p. 467, #8

Puzzle of the week (optional!)

Conditional expectation as the best mean-square predictor. Suppose we would like to predict the value of a random variable Y given some information. For example, suppose we observe some other random variable X whose joint distribution with Y is assumed to be known. We will aim to predict the value of Y by a function of X, say $g(X)$. Once the value x of X is known, the value $g(x)$ of $g(X)$ is used as a prediction of the unknown value of Y.

One measure of the goodness of the predictor $g(X)$ is the mean-squared error

$$\text{MSE}(g(X)) = E\big[(Y - g(X))^2\big].$$

Show that $g(X) = E\big(Y|X\big)$ minimizes $\text{MSE}(g(X))$. To do this, use average conditional expectation given $X = x$, expand the square in the conditional expectation, use that $g(x)$ is some constant c for each fixed x, and minimize the appropriate expression by differentiating with respect to c and setting equal to 0.