M362K (56310), Homework #6

Due: 12:30pm, Thursday, Mar. 03

Instructions: Please show all your work, not only your final answer, in order to receive credit. Please keep answers organized in the same order the problems have been assigned.

Random variables (cont'd) (3.1)

1. Pitman, p. 159, #8
2. Pitman, p. 159, #10
3. Pitman, p. 160, #15
4. Pitman, p. 161, #21

Expectation (3.2)

5. Pitman, p. 182, #2
6. Pitman, p. 182, #3
7. Pitman, p. 182, #4
8. Pitman, p. 182, #6
9. Pitman, p. 182, #7
10. Pitman, p. 182, #10
11. Pitman, p. 183, #14
12. Pitman, p. 183, #21

Puzzle of the week (optional!)

Expectation and the mean, median, and mode of a distribution. Suppose we would like to predict the value of a random variable X. Let b be our prediction. Now suppose we will pay a cost $L(x,b) \geq 0$ if X actually has value x (typically, $L(x,b)$ is known as a *loss function*). So, to minimize our cost we should pick b so that the *expected loss* (also called *risk*)

$$r(b) = E[L(X,b)]$$

is minimized. The best prediction b therefore depends on our choice of loss function. Show that

i. if $L(x,b) = \begin{cases} 0 & \text{if } X = b \\ 1 & \text{if } X \neq b \end{cases}$, then b is a *mode* of the distribution of X (a mode is a value x such that $P(X = x)$ is maximal, not necessarily unique)

ii. if $L(x,b) = |X - b|$, then b is a *median* of the distribution of X (a median is a value x such that $P(X \leq x) \geq 1/2$ and $P(X \geq x) \geq 1/2$, not necessarily unique)

iii. if $L(x,b) = (X - b)^2$, then b is the *mean* of the distribution of X (i.e., $b = E(X)$).