

M362K (56310), Sample Midterm 1

Instructions: Please show all your work, not only your final answer, in order to receive credit. Please keep answers organized in the same order the problems have been assigned.

Recommended time: 1 hour

1. (Pitman, p. 490, #1) Ten dice are rolled. Five dice are red and five are green. Write down numerical expressions for:

- a) The probability of the event that exactly four of the ten dice are sixes.
- b) The probability of the event that exactly two of the red dice are sixes and exactly three of the green dice show even numbers.
- c) The probability that there are the same number of sixes among the red dice as among the green dice.
- d) The probability that there are strictly more sixes among the red dice than among the green dice.

2. (Pitman, p. 490, #4(a,b)) Let A , B , and C be events which are mutually independent, with probabilities a , b , and c . Let N be the random number of events which occur.

- a) What is the event $(N = 2)$ in terms of A , B , and C ?
- b) What is the probability of this event in terms of a , b , and c ?

3. (Pitman, p. 491, #1) **Coin spinning.** I have two coins. One shows heads with probability $1/10$ when spun. The other shows heads with probability $1/2$. Suppose you pick one of my two coins at random and spin it twice. Find:

- a) $P(\text{heads on first spin})$;
- b) $P(\text{heads on second spin})$;
- c) $P(\text{heads on both spins})$;
- d) the probability that the coin is the $1/2$ coin given heads on both spins.

4. (Pitman, p. 491, #2) **True or false.** A student answers a set of 100 true/false questions by answering 36 questions correctly, and guessing the other 64 at random.

- a) If the pass mark is 70 questions correct, what is the student's chance of passing? Give your answer (by using an approximation) as a decimal correct to two places.
- b) Another student also knows 36 correct answers and guesses the rest at random. What is the chance that just one of these two students passes?

5. (Pitman, p. 491, #3) **Rare white balls.** A box contains 998 black and 2 white balls. Let X = the number of whites in 500 random draws *with* replacement from this box. Calculate:

- a) $P(X = 1)/P(X = 2)$ exactly, and approximately using the Poisson approximation;
- b) $P(X = 1 \text{ given } X = 1 \text{ or } 2)$ exactly, and approximately using the Poisson approximation;
- c) repeat (b) assuming draws without replacement (only compute exactly).

6. (Pitman, p. 492, #2) A box contains 5 tickets. An unknown number of them are red, the rest are green. Suppose that to start off with you think there are equally likely to be 0, 1, 2, 3, 4, or 5 red tickets in the box.

- a) Three tickets are drawn from the box with replacement between draws. The tickets drawn are red, green, and red. Given this information, what is the chance there are actually 3 red tickets in the box?
- b) What would your answer to (a) be if you knew the draws were made without replacement?
- c) Now suppose that there actually are 3 red tickets in the box. Suppose we sample tickets from the box 3 times without replacement. Let $R = \#$ of red tickets drawn and $G = \#$ of green tickets drawn. What is the joint distribution of the random variables R and G ?
- d) Are R and G independent? Show why or why not.