M362K (56310), Sample Midterm 2

Instructions: Please show all your work, not only your final answer, in order to receive credit. Please keep answers organized in the same order the problems have been assigned.

Recommended time: 1 hour, 15 minutes

1. (Pitman, p. 490, #3) A student takes a multiple choice examination where each question has 5 possible answers. He works a question correctly if he knows the answer, otherwise he guesses at random. Suppose he knows the answer to 70% of the questions.

a) What is the probability that on a question chosen at random the student gets the correct answer?

b) Given that the student gets the correct answer to this question chosen at random, what is the probability that he actually knew the answer?

c) Suppose there are 20 questions on the examination. Let N be the number of questions that the student gets correct. Find $E(N)$.

d) Find $SD(N)$.

2. (Pitman, p. 493, #8(a)) Let U_1 and U_2 be two independent uniform $[0, 1]$ random variables. Let $X = \min(U_1, U_2)$ where $\min(u_1, u_2)$ is the smaller of two numbers u_1 and u_2. Find the cdf F_X and the probability density function f_X of X.

3. (Pitman, p. 494, #3) A die has one spot painted on one face, two spots painted on each of the two faces, and three spots painted on each of three faces. The die is rolled twice.

a) Calculate the distribution of the sum S_2 of the numbers on the two rolls. Display your answer in a table.

b) Calculate the numerical value of $E(S_2)$ in two different ways to check your answer to (a).

c) Calculate the standard deviation of S_2.

4. (Pitman, p. 494, #4) Suppose the average family income in a particular area is \$10,000.

a) Find an upper bound for the fraction of families in the area with incomes over \$50,000.

b) Find a smaller upper bound than in (a), given that the standard deviation is \$8000.

c) Do you think the normal approximation would give a good estimate for the fraction in question?

5. (Pitman, p. 494, #6) Telephone calls arrive at an exchange at an average rate of one every second. Find the probabilities of the following events, explaining briefly your assumptions.

a) No calls arriving in a given five-second period.

b) Between four and six calls arriving in the five-second period.

c) Between 90 and 110 calls arriving in a 100-second period. (Give answer as a decimal.)

6. (Pitman, p. 496, #7) A particle counter records two types of particles, Types 1 and 2. Type 1 particles arrive at an average rate of 1 per minute, Type 2's at average rate of 2 per minute. Assume these are two independent Poisson processes. Give numerical expressions for the following probabilities.

a) Three Type 1 particles and four of Type 2's arrive in a two-minute period;

b) the total number of particles of either type in a two-minute period is 5;

c) the fourth particle arrives in the first 5 minutes;

d) the first particle to arrive is of Type 1;

e) the second particle of Type 1 turns up before the third of Type 2.

7. (Pitman, p. 496, #8) Consider the average $\bar{X}_n = (X_1 + X_2 + \cdots + X_n)/n$ of n independent random variables, each uniformly distributed on $[0, 1]$. Find n so that $P(\bar{X}_n < 0.51)$ is approximately 90%.

8. (Pitman, p. 496, #10) Suppose 10 dice are shaken together and rolled. Any that turn up six are set aside. The remaining dice are shaken and rolled again. Any of these that turn up six are set aside. And so on, until all the dice show six. Let N be the number of times the dice are shaken and rolled. To illustrate, if after the first roll of 10 dice, 7 non-sixes remain, and after the second roll of these 7 dice 2 non-sixes remain, and after the third roll of these 2 dice no non-sixes remain, then $N = 3$.

a) Give the distribution of N. *[Hint: Consider the number of times W_i die i is rolled. Now express N as a function of the W_i.]*

b) Let T be the total number of individual die rolls. To illustrate, $T = 10 + 7 + 2 = 19$ for the outcome described above. Give the distribution of T. *[Hint: Express T as a function of the W_i.]*

c) Let L be the number of dice shaken on the last roll. To illustrate, $L = 2$ for the outcome described above. Give the distribution of L. *[Hint: First consider the joint distribution of N and L.]*