M408D (54690/95/00), Sample Midterm #2

Multiple choice questions (5 points each)

See last two pages.

Question #1 (20 points)

Define

$$\boldsymbol{u} = \langle 0, -1, 2 \rangle$$
$$\boldsymbol{v} = \langle 3, 4, 5 \rangle$$
$$\boldsymbol{w} = \langle -3, 7, 1 \rangle.$$

a) What is $\boldsymbol{u} \times \boldsymbol{v} \times \boldsymbol{w}$?

b) Find a vector equation for the plane parallel to $\boldsymbol{v}$ and $\boldsymbol{w}$ that passes through the point $\boldsymbol{u}$.

c) What is the scalar projection $\text{comp}_{\boldsymbol{w}}\boldsymbol{u}$? [Hint: Remember that the scalar projection of $\boldsymbol{a}$ onto a unit vector $\boldsymbol{e}$ is $|\boldsymbol{a} \cdot \boldsymbol{e}|$. Now take $\boldsymbol{e} = \boldsymbol{b}/|\boldsymbol{b}|$ to find the scalar projection onto some arbitrary vector $\boldsymbol{b}$.]

Question #2 (20 points)

Consider the quadric surface given by

$$6z = x^2 + 4y^2 - 1.$$

a) Find the trace of the surface in the plane $2y - z - 1 = 0$. What kind of curve is this?

b) Does the curve $\boldsymbol{r}(t) = \langle \sin t, \frac{1}{2}\cos t, t \rangle$ lie on the surface $6z = x^2 + 4y^2 - 1$? If not, at what point $P(x_0, y_0, z_0)$ does it intersect it?

Question #3 (20 points)

Define

$$\boldsymbol{r}'(t) = \langle -2\sin t, 2\cos t, 0 \rangle$$

and suppose $\boldsymbol{r}(0) = \langle 1, 2, 3 \rangle$.

a) What is $\boldsymbol{r}(t)$?

b) Find the unit tangent vector $\boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|}$.

c) What is the curvature $\kappa(t) = \frac{|\boldsymbol{T}'(t)|}{|\boldsymbol{r}'(t)|}$ of the curve at the point $P(1, 2, 3)$?

d) Determine a normal vector to the curve at $P(1, 2, 3)$. [Hint: Remember that $\boldsymbol{T}$ and $\boldsymbol{T}'$ are perpendicular to each other!]

Question #4 (15 points)

Consider the polar equation

$$r = \sin\theta.$$

a) Rewrite this equation in Cartesian coordinates and graph the curve. That is, find the corresponding equation in x and y. What conic section is this?

b) The polar curve $r = 2\theta$ lies further away from the origin than $r = \sin\theta$ since for every θ $2\theta \geq \sin\theta$. Find the area that lies between these two curves between the angles $\theta = 0$ and $\theta = \pi/2$ using the formula $A = \frac{1}{2}\int_{\theta_0}^{\theta_1} |f^2(\theta) - g^2(\theta)|d\theta$ for the area between two polar functions.

This print-out should have 5 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

CalC11a16s
001 10.0 points

Determine a Cartesian equation for the curve given in parametric form by

$$x(t) = 4\ln(4t), \quad y(t) = \sqrt{t}.$$

1. $y = \frac{1}{4}e^{x/2}$

2. $y = \frac{1}{4}e^{x/4}$

3. $y = \frac{1}{4}e^{4/x}$

4. $y = \frac{1}{2}e^{8/x}$

5. $y = \frac{1}{2}e^{x/8}$ **correct**

6. $y = \frac{1}{2}e^{x/4}$

CalC13a30a
002 10.0 points

Find an equation for the set of all points in 3-space equidistant from the points

$$A(-1, -4, 1), \qquad B(3, -3, 4).$$

1. $x + 4y - 3z - 8 = 0$

2. $4x + y + 3z + 8 = 0$

3. $4x + y + 3z - 8 = 0$ **correct**

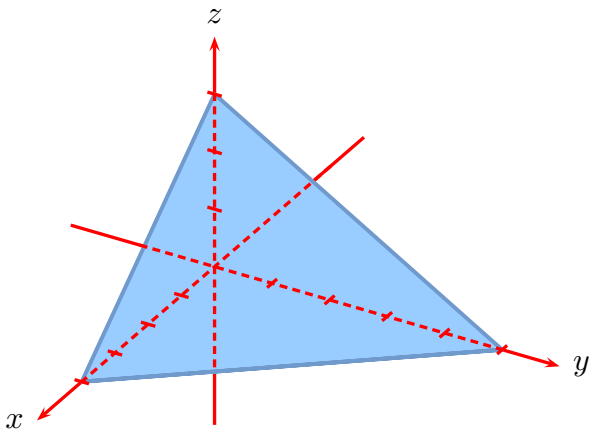
4. $3x + y + 4z + 8 = 0$

5. $x - 3y + 4z + 8 = 0$

6. $3x - 4y - z - 8 = 0$

CalC13e02b
003 10.0 points

Which equation has the surface



as its graph in the first octant?

1. $\frac{x}{5} + \frac{y}{4} + \frac{z}{3} = 1$

2. $\frac{x}{4} + \frac{y}{5} + \frac{z}{3} = 1$ **correct**

3. $\frac{x}{3} + \frac{y}{5} + \frac{z}{4} = 1$

4. $\frac{x}{4} + \frac{y}{3} + \frac{z}{5} = 1$

5. $\frac{x}{5} + \frac{y}{3} + \frac{z}{4} = 1$

6. $\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 1$

CalC13c31a
004 10.0 points

Find the vector projection of **b** onto **a** when

$$\mathbf{b} = \langle -1, 3 \rangle, \qquad \mathbf{a} = \langle 2, 1 \rangle.$$

1. vector proj. $= \frac{3}{\sqrt{5}}\langle -1, 3 \rangle$

2. vector proj. $= \frac{1}{5}\langle 2, 1 \rangle$ **correct**

3. vector proj. $= \frac{1}{5}\langle -1, 3 \rangle$

4. vector proj. $= \frac{3}{\sqrt{5}}\langle 2, 1 \rangle$

5. vector proj. $= \frac{3}{5}\langle 2, 1 \rangle$

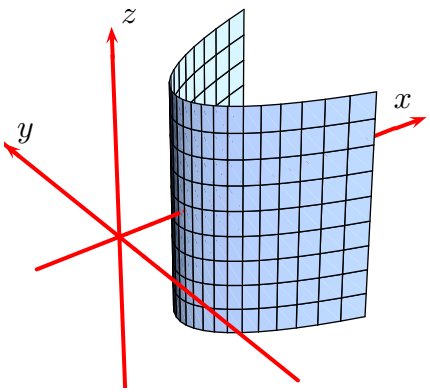
6. vector proj. $= \frac{1}{\sqrt{5}}\langle -1, 3 \rangle$

CalC13f04d
005 10.0 points

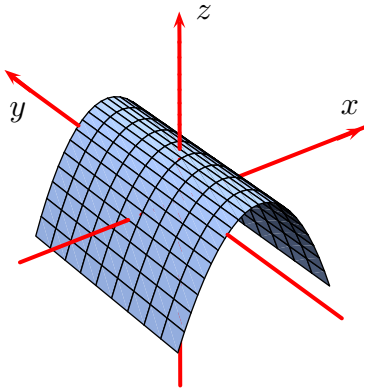
Which one of the following is the graph of the equation

$$x^2 + z = 1\,?$$

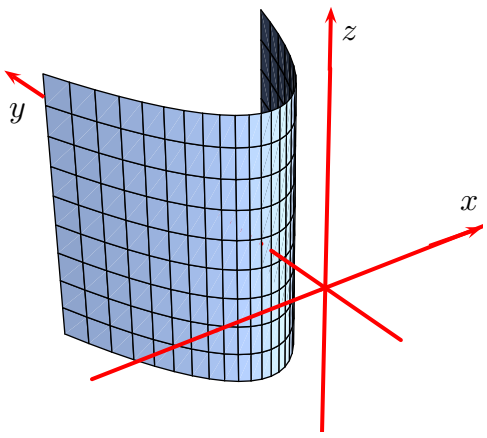
1.



2. **correct**



3.



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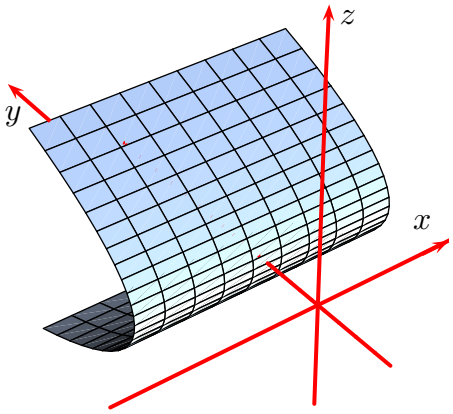

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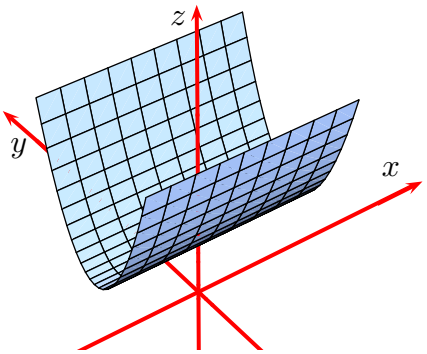

6.