M427L (55200), Quiz #2 Solutions

Question #1 (5 pts.)

Let P be the plane which contains the point (0, 1, 2) and the line given by the vector equation $\boldsymbol{r}(t) = (4-3t)\boldsymbol{i} + 2t\boldsymbol{j} + (1-t)\boldsymbol{k}$, $t \in \mathbb{R}$. Find the distance d between this plane P and the point $E = (6, -4, 3)$. (Hint: If you've forgotten the formulas, try rederiving them using a picture!)

Solution: [*Question based on p. 62, #16, p. 63, #27, 28, and problem done in lecture.*] First let us find the equation for the plane P. We already know that the point $\boldsymbol{p}_0 = (0, 1, 2) \in P$ and that the line $\boldsymbol{r}(t)$ lies in the plane as well. Therefore, the vectors $\boldsymbol{a} = \boldsymbol{r}(0) - \boldsymbol{r}(1) = (3, -2, 1)$ and $\boldsymbol{b} = (0, 1, 2) - \boldsymbol{r}(1) = (-1, -1, 2)$ are parallel to the plane and a normal vector to P is given by

$$\boldsymbol{n} = \boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & -2 & 1 \\ -1 & -1 & 2 \end{vmatrix} = -3\boldsymbol{i} - 7\boldsymbol{j} - 5\boldsymbol{k}.$$

Therefore, P satisfies the vector equation $\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{p}_0) = 0$.

We now use the formula

$$d = \frac{|\boldsymbol{n} \cdot (\boldsymbol{c} - \boldsymbol{p}_0)|}{\|\boldsymbol{n}\|} = \frac{|(3, 7, 5) \cdot (6, -5, 1)|}{\sqrt{83}} = \frac{12}{\sqrt{83}}$$

for the distance between the point $\boldsymbol{c} = \overline{OE} = (6, -4, 3)$ and the plane P.

Question #2 (3 pts.)

Consider the surface given in spherical coordinates by

$$\rho^2 \sin(2\phi) \cos(\theta) = 2.$$

Using the double-angle formula $2 \sin\phi \cos\phi = \sin(2\phi)$, express this surface in terms of Euclidean coordinates (x, y, z). What does the surface look like? (Again, if you've forgotten the formulas for converting between coordinate systems, rederive them!)

Solution: [*Question based on definition of spherical coordinates.*] The change of coordinates from spherical coordinates to Euclidean coordinates is

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ x &= \rho \cos\phi. \end{aligned}$$

The surface can therefore be written as $(\rho \sin(\phi) \cos(\theta))(\rho \cos\phi) = 1$, i.e., $xz = 1$. This describes a hyperbola in the xz-plane which is extended in the y-direction.