M427L (55200), Sample Midterm #1 Questions

Below are some sample midterm questions. Please note that the intent of these is to help prepare for the exam, and that actual exam questions will not merely be modifications of these problems. Consult your HW, quizzes, lecture notes, and book for additional sources of material to review. Finally, to obtain the full effect of an exam please complete these problems under time-pressure (75 minutes or less).

1. Define the vectors

$$\begin{aligned} \boldsymbol{a} &= (4, 2, 0) \\ \boldsymbol{b} &= (1, -3, 5) \\ \boldsymbol{c} &= (-2, 2, 1). \end{aligned}$$

   a) What is the volume of the parallelpiped whose edges are given by the vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$?

   b) What is the cosine of the angle between the vectors $\boldsymbol{b}$ and $\boldsymbol{c}$?

   c) Determine the equation for the plane parallel to the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ that passes through the point $(1, 1, 1)$ (write in the form $\boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r_0}) = 0$).

2. Define

$$h(x, y) = 2x^3 + xy^2 + 5x^2 + y^2.$$

   a) Find all critical points of h.

   b) Classify all critical points (i.e., determine if they are local maxima, minima, or saddle points) by using the second derivative test. To begin, compute the Hessian of $h(x, y)$.

   c) What are the absolute maximum and minimum values of h on the domain $D_1 = \{(x, y): 0 \leq x \leq 4, 0 \leq y \leq 5\}$?

3.

   a) Find the equation of the tangent plane to the surface $x^2 - e^{xy} + z^2 = 1$ at $(1, 0, 1)$.

   b) Find the equation of the line perpendicular to the surface in part (a) at the point $(1, 0, 1)$.

   c) What is the distance between the plane found in part (a) and the origin?

4.

   a) Determine the maximum and minimum values of the function

$$f(x, y) = xy^2$$

   on the ellipse $x^2 + \frac{1}{4}y^2 = 1$.

   b) Moving clockwise along the ellipse, is the function increasing or decreasing at the point $(0, -2)$?

5. Define the function

$$f(x, y, z) = x \sin(yz).$$

a) Determine $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

b) Does f satisfy Laplace's equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$?

c) Suppose

$$\begin{aligned} x(s, t) &= \cos(s^2 + t) \\ y(s, t) &= e^{-2st} \\ z(s, t) &= s^3 - 2st^2 + 4. \end{aligned}$$

Determine $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$.

d) Find the directional derivative of f at the point $(2, 0, 1)$ in the direction of the vector $\boldsymbol{v} = -\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$. How does this value compare to the maximal rate of increase at $(2, 0, 1)$?

6. Consider the vector-valued function

$$\boldsymbol{r}(t) = \left(2e^t, 3t^2, te^{4t}\right).$$

a) What is $\boldsymbol{r}''(t)$?

b) At what point does the curve $\boldsymbol{r}(t)$ intersect the surface $16z = x^4$?

c) Find the tangent vector to the curve at the point $(2, 0, 0)$.

7. Does the limit

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^2 y - z}{x^4 + y^2 + z}$$

exist? Show why or why not.