M427L (55200), Sample Midterm #2 Questions

Below are some sample midterm questions. Please note that the intent of these is to help prepare for the exam, and that actual exam questions will not merely be modifications of these problems. Consult your HW, quizzes, lecture notes, and book for additional sources of material to review. Finally, to obtain the full effect of an exam please complete these problems under time-pressure (75-90 minutes or less).

1.

a) Draw the vector field $\boldsymbol{F}(x,y)=(y,-x)$.

b) Find an expression for a flow line $\boldsymbol{c}(t)$ for $\boldsymbol{F}$ by taking a good guess.

c) Do the same for $\boldsymbol{F}(x,y)=(2x,-y)$.

2.

a) Find the arc length of $\boldsymbol{c}(t)=\left(2t,t^2\right)$, $0\le t\le 1$.

b) Find the arc length of $\boldsymbol{c}(t)=\left(2\sin^2 t,\sin^4 t\right)$, $0\le t\le \pi/2$.

c) Explain your answers to parts (a) and (b).

3.

a) Evaluate the following integral over the region D bounded by the lines $x+y=1$, $x+y=-1$, $x-y=1$, and $x-y=-1$:

$$\int\int_D 3(x+y)e^{x-y}\,dx\,dy.$$

b) Evaluate

$$\int_0^1\int_0^x \sin y^2\,dy\,dx.$$

c) Find the mass of a wall described by $0\le y\le -x^2-2x+3$, $-3\le x\le 3$, having mass density $\rho(x,y)=2|y|+3$.

4. Let $\boldsymbol{F}(x,y)=e^x\cos 3y\boldsymbol{i}-3e^x\sin 3y\boldsymbol{j}$.

[Hint: Use shortcuts to quickly find answers to the following questions.]

a) Find an $f(x,y)$ such that $\nabla f=\boldsymbol{F}$ for all (x,y).

b) Evaluate $\int_{\boldsymbol{c}}\boldsymbol{F}\cdot d\boldsymbol{s}$ for the path $\boldsymbol{c}(t)=(\cos t,\sin t)$, $0\le t\le\pi$.

c) Compute $\operatorname{div}\boldsymbol{F}$ and $\operatorname{curl}\boldsymbol{F}$.

5.

a) A particle moves in a path $\boldsymbol{c}(t) = (2t, 3t, t)$ in a force field $\boldsymbol{F} = (2x, 2y, 4z)$. What is the work done by $\boldsymbol{F}$ in the time interval $0 \le t \le 5$?

b) With the same $\boldsymbol{c}(t)$ and $f(x, y, z) = xyz$, what is $\int_{\boldsymbol{c}} f ds$?

6. A hole of radius 1/2 is drilled through the axis of symmetry of the hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$.

a) Write (but do not compute) an integral for the volume of the remaining piece in cartesian coordinates.

b) Write (but do not compute) an integral for the volume of the remaining piece in cylindrical coordinates.

c) Compute the volume.

7.

a) Use the vector field $\boldsymbol{V} = \nabla f$ with $f(x, y, z) = c_1 x + c_2 y + c_3 z$ to explain the meaning of divergence. For this particular choice of vector field, what is $\nabla \times \boldsymbol{V}$?

b) Use the vector field $\boldsymbol{V}(x, y) = -\omega y \boldsymbol{i} + \omega x \boldsymbol{j}$ with ω constant to explain the meaning of curl. For this particular choice of vector field, what is $\nabla \cdot \boldsymbol{V}$?

c) Are the divergence and curl at a point local quantities or do they depend on the entire vector field?