IMPORTANT LIMITS

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1. Important Limits

(1) If $x > 0$, then

$$x^{1/n} \longrightarrow 1 \quad \text{as} \quad n \longrightarrow \infty$$

(2) If $|x| < 1$, then

$$x^n \longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty$$

(3) For each $\alpha > 0$,

$$\frac{1}{n^\alpha} \longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty$$

(4) For each real x,

$$\frac{x^n}{n!} \longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty$$

(5)

$$\frac{\ln n}{n} \longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty$$

(6)

$$n^{1/n} \longrightarrow 1 \quad \text{as} \quad n \longrightarrow \infty$$

(7)

$$\left(1 + \frac{x}{n}\right)^n \longrightarrow e^x \quad \text{as} \quad n \longrightarrow \infty$$

2. Whose Bigger

Here is a list of whose bigger than who.

$$\ln n < n < n^2 < \cdots < n^{100} < \cdots < 2^n < 3^n < \ldots 100^n < \ldots$$
$$< n! < n!2^n < n^n < 3^n n! < n^{2n} < \ldots$$

There is no biggest, so our list goes on and on and on, like the Energizer Bunny.

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If a sequence a_n comes before b_n in the list, then

$$\lim_{n \longrightarrow \infty} \frac{a_n}{b_n} = 0$$

As an example, n^{100} comes before 2^n, so

$$\lim_{n \longrightarrow \infty} \frac{n^{100}}{2^n} = 0$$

Likewise, if a_n comes before b_n, then

$$\frac{a_n}{b_n} \longrightarrow \infty \quad \text{as} \quad n \longrightarrow \infty$$

This is not a way to do any problems. This is to help you know the answer before you actually do the problem. You still need to use the techniques of the book to show the limit.

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