INTEGRATION TECHNIQUES

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I have decided to put to paper what was reviewed in discussion on Thursday.

1. Integration by Parts

We use I.L.A.T.E. when we perform integration by parts. Recall, that

$$\int u dv = uv - \int v du \tag{1}$$

or equivalently, we have

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx \tag{2}$$

You can always remember this by taking the product rule

$$\frac{d}{d}\Big(f(x)g(x)\Big) = f'(x)g(x) + f(x)g'(x)$$

Then integrating

$$\int \frac{d}{dx}\Big(f(x)g(x)\Big)dx = \int f'(x)g(x)dx + \int f(x)g'(x)dx$$

which becomes

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

Now, I.L.A.T.E. tells you what choice you should make for u in Equation (1). We have

I	inverse
L	logarithmic
A	algebraic
T	trigonometric
E	exponential

So that if we had

$$\int xe^x dx$$

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we see that x is algebraic and e^x is exponential. So

$$u = x \quad dv = e^x dx$$
$$du = dx \quad v = e^x$$

If we had

$$\int x \sin^{-1} x dx$$

we see that x is algebraic and $\sin^{-1}(x)$ is an inverse, so

$$u = \sin^{-1} x \quad dv = xdx$$
$$du = \frac{1}{\sqrt{1-x^2}}dx \quad v = \frac{x^2}{2}$$

2. Partial Fraction Decomposition

The idea of partial fraction decomposition is to split something like

$$\frac{1}{(x-a_1)(x-a_2)(x-a_3)\dots(x-a_n)}$$

into

$$\frac{A_1}{x-a_1} + \frac{A_2}{x-a_2} + \frac{A_3}{x-a_3} + \dots + \frac{A_n}{x-a_n}$$

or if you have an irreducible quadratic factor $Q(x)$, the piece associated with this term would be

$$\frac{Ax+B}{Q(x)}$$

Notice that if every polynomial had all of its roots in the real numbers, then we would never have to consider this quadratic piece. So part fraction decomposition over the complex numbers is much simpler (practically everything is simpler over the complex numbers, which is strange since we call them "complex").

So I will work an example:

Example 2.1. Consider

$$\frac{1}{(x-2)(x+2)x}$$

We want

$$\frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$$

Now, we have

$$\frac{A(x-2)(x+2) + Bx(x+2) + Cx(x-2)}{x(x-2)(x+2)} = \frac{1}{x(x-2)(x+2)}$$

So that since the denominator is equal in both fractions, the numerator must be equal. So that

$$A(x-2)(x+2) + Bx(x+2) + Cx(x-2) = 1$$

Now, this holds for all x. So we choose a few, as to obtain some algebraic equations:

$$\begin{aligned} x &= 0, & -4A &= 1 \\ x &= 2, & 8B &= 1 \\ x &= -2, & -8C &= 1 \end{aligned}$$

So that we have solved for A, B, and C, and we get

$$\frac{-\frac{1}{4}}{x} + \frac{\frac{1}{8}}{x-2} + \frac{-\frac{1}{8}}{x+2}$$

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