MATH 316 (RUSIN) TEST 2, Mar 30 2012. Name & UT EID: ____________________

Please put everything you want me to see on these pages. If you need additional pages, please make sure your name and UT EID are on every additional sheet. You must explain and show your work. All questions will be weighted equally.

1. Four identical coins are tossed, one after another.

1a. What is the probability that they will all come up "heads"?

1b. What is the probability that at there will be exactly three "heads" and one "tail"?

1c. Your answer in 1b should be greater than your answer in 1a. So does this mean that after three "heads" have already been tossed, you are more likely to toss "tails" next?

2. Pat is applying to several medical schools and has consulted the schools to find out what the chances of being admitted are. Pat discovers Johns Hopkins medical school accepts 50% of its applicants, and Harvard University medical school accepts 40% of its applicants.

2a. Assume that the acceptances into the two schools are independent events. What is the probability that Pat will be accepted into at least one of these two schools?

2b. In reality, Harvard and Hopkins are usually interested in the same students — a student accepted at one of them has a higher-than-average chance of being accepted at the other. Does this fact increase or decrease Pat's chances of being accepted by at least one of the two schools? (You may wish to illustrate with a numerical example.)

3. You will be eligible to collect Social Security benefits on your 67th birthday; if you turn 18 today, that's exactly 49 years in the future. To invest for your retirement, you might buy a broad variety of U.S. stocks because there is an annual percentage gain x in the value of such an investment; historically, x has acted like a random value with a mean of about $\mu = 9$ percent per year and a standard deviation of about $\sigma = 21$ percent per year.

Assume that the values of x from year to year are uncorrelated, so that the net effect of investing for the next 49 years is the same as picking 49 values of x randomly. Then what is the probability that your stocks will lose value over that much time?

4. In Chapter 14 we discussed statistical inference and encountered a lot of mumbo-jumbo like this: "The average body-mass index (BMI) of young Americans is 26.8 ± 1.9 with 95% confidence." Use the terms of Chapter 14 to explain what this means. (For example, why is there any uncertainty at all — can't the researchers do simple arithmetic and average all the BMI values? What does it mean to measure "confidence"? Can't they just replace the 1.9 by 2.0 and say what the average BMI is, with 100% confidence?)

5. You have data on an SRS of n recent graduates from the University of Texas that shows how long each person took to complete their degree. We want to use this SRS to estimate the mean, μ, of the statistical variable x = time-to-graduation, over all UT graduates. We would like to report our estimate to the press along with an 80% confidence interval.

While the study is being completed, a student named Chris Slacker finally manages to graduate after a record 23 years of continuous enrollment, so is now part of the population of all UT graduates, but is not part of the SRS that you have already selected.

Explain which of the following quantities change, and whether they get larger or smaller: n, μ, $\bar{x}$, the standard deviation σ, and the width of the 80% confidence level.

6. The Rusin Aspirin Company (RAC) makes the claim that taking an aspirin before taking a Statistics exam will substantially improve your score on the exam. Let's test this claim with the help of 1000 students taking Math 316. One hundred of them, selected at random, are given an aspirin before this exam. (The others are given a placebo. Those grading the exam have no idea which students got the aspirin.)

For each student we then compute the amount x that their exam 2 score improved from exam 1. This x is observed to follow an approximately Normal distribution. The average for all 1000 students is $\mu = +20$, and the standard deviation is $\sigma = 25$.

For the students who took the aspirin first, it is found that the average change in their test scores was $\bar{x} = +25$, which appears to support the claim made by RAC; but is this finding significant at say the $\alpha = 0.10$ level? At the $\alpha = 0.01$ level? Explain.

7. NASA wishes to determine the expected lifetime of rats in space. It is very expensive to send a rat into space so only 24 rats were included in the experiment. (A careful process was followed to make sure the rats were selected without any bias.)

The lifetime x (measured in days) for each of the rats was noted, and NASA computed the average $\bar{x} = 595$ and the standard deviation $s = 84$ of this sample. So what should NASA report as its estimate of the average lifespan of a rat in space, if it wants to give a 95% confidence interval?

Here are some of the entries from Table A:

z	*Area*	z	*Area*	z	*Area*	z	*Area*
−5.0	.0000	−1.2	.115	0.0	.500	1.3	.903
−4.0	.0001	−1.1	.136	0.1	.540	1.4	.919
−3.0	.001	−1.0	.159	0.2	.579	1.5	.933
−2.5	.006	−0.9	.184	0.3	.618	1.6	.945
−2.2	.014	−0.8	.212	0.4	.655	1.7	.955
−2.0	.023	−0.7	.242	0.5	.691	1.8	.964
−1.9	.029	−0.6	.274	0.6	.726	1.9	.971
−1.8	.036	−0.5	.309	0.7	.758	2.0	.977
−1.7	.045	−0.4	.345	0.8	.788	2.2	.986
−1.6	.055	−0.3	.382	0.9	.816	2.5	.994
−1.5	.067	−0.2	.421	1.0	.841	3.0	.999
−1.4	.081	−0.1	.460	1.1	.864	4.0	.9999
−1.3	.097	−0.0	.500	1.2	.885	5.0	1.0000

And here are some of the entries from Table C:

Degrees	Conf		Level	C
of Freedom	80%	90%	95%	99%
1	3.08	6.31	12.7	63.7
2	1.89	2.92	4.30	9.93
11	1.36	1.80	2.20	3.11
12	1.36	1.78	2.18	3.06
23	1.32	1.72	2.07	2.81
24	1.32	1.71	2.06	2.80
25	1.31	1.71	2.06	2.79
z^*	1.28	1.65	1.96	2.58
One-sided P	0.10	0.05	0.025	0.01
Two-sided P	0.20	0.10	0.05	0.02