M328K – Rusin – HW1 – Due Thursday, Jan 26 2017

1. Show that the number 123,456,789,876,543,201,234,567 is not a perfect square. (Hint: According to the "Division Algorithm" theorem, for every integer a there exists an integer q such that either

$a = 5q$ or $a = 5q+1$ or $a = 5q+2$ or $a = 5q+3$ or $a = 5q+4$

What does that tell you about a^2?)

2. Show me how well you can write a proof by induction by finding (and proving!) a formula for the product of the first n powers of 2. For example, the product of the first three powers of 2 is $2 \cdot 4 \cdot 8 = 64$.

3. Prove that any amount of postage over $1 can be paid for with a combination of 7-cent stamps and 11-cent stamps. (Postage is always a positive integer number of cents.)

4. Do there exist positive integers a, b, c for which $a \mid (bc)$ but $a \nmid b$ and $a \nmid c$?

5. Show that if a and b are integers with $a \mid b$ then $a^2 \mid b^2$.

6. Recall that a set of real numbers is called *well-ordered* if every non-empty subset of it has a smallest element. Is the set of non-negative rational numbers well-ordered?

Challenge: I mentioned in class that it can be difficult to know whether an integer n can be represented as a sum of three cubes. Can you find three integers x, y, z for which $x^3 + y^3 + z^3 = 12$? How about 13?