

M365C (Rusin) HW6 – due Thursday, Oct 17 2019

1. The series $\sum_{n \geq 0} x^{(n^2)}$ is an example of a *theta function*. For what real numbers x does this power series converge?
2. You know that for each individual $x \in (-2, 2)$, the series $\sum_{n \geq 0} (2^{-n})x^n$ converges to $2/(2-x)$. But view each of these functions as elements of $C^0[0, 1]$; in that sense, does the series $\sum_{n \geq 0} (2^{-n})x^n$ converge to $2/(2-x)$?
(You might also think about why I didn't ask about the simpler series $\sum x^n$.)
3. Show, directly from the definition of continuity, that the function $f(x) = \sqrt{|x|}$ is continuous at $x = 0$. Similarly show that the function $g(x) = 1/x$ is continuous at each point $x = a$ in its domain (i.e. for each $a \neq 0$).
4. What functions $f : \mathbf{R} \rightarrow X$ are continuous, when X is a discrete metric space? What functions $g : X \rightarrow \mathbf{R}$ are continuous?
5. Suppose X is a metric space, and A and B are closed subsets of X whose union is all of X . Show that if $f : X \rightarrow Y$ is a function which is continuous on A and continuous on B then it is continuous on X . Give an example to show that this need not be true when A is not required to be closed.