1. Find the equation of the plane that passes through the points \((1, 2, 2)\) and \((-1, 1, 3)\) and is parallel to the line \(x = 1 + 2t, y = 4 - t, z = 3t\).

The normal to this plane must be perpendicular to the vector \((1, 2, 2) - (-1, 1, 3) = (2, 1, -1)\) and also normal to the direction vector \(2, -1, 3\) of the line, hence must point in the direction of their cross product, which is \((2, -8, -4)\), so the equation of the plane is \(2x - 8y - 4z = D\) for some constant \(D\). Plugging in either of the points gives \(D = -22\), so our plane is \(-2x + 8y + 4z = 22\), or \(-x + 4y + 2z = 11\).

2. Let \(f(x) = \sin(x^3)\). Find the 99th derivative of \(f\) evaluated at 0. That is, find \(f^{(99)}(0)\).

Since \(\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}\), \(\sin(x^3) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{6k+3}}{(2k+1)!}\). The coefficient of \(x^{99}\) is \(1/33!\), so the 99th derivative at \(x = 0\) is \(99!/33!\).

3. Find the point on the ellipse \(\frac{x^2}{4} + \frac{y^2}{9} = 1\) that is farthest from the line \(2x + y = 10\).

Since the line has slope -2, the nearest and farthest points must be where the tangent to the ellipse has slope -2. By implicit differentiation, \(\frac{dy}{dx} = -\frac{9x}{4y}\). Setting this equal to -2, we get \(y = 9x/8\), and plugging into the equation of the ellipse gives \(x = \pm 8/5, y = \pm 9/5\). The plus signs are for the closest point, and the minus signs are for the farthest point, namely \((-8/5, -9/5)\).
4. Let $C_1$ be the solid cylinder in 3-dimensional space consisting of all points whose distance from the $x$-axis is not greater than 6. Let $C_2$ be the solid cylinder consisting of all points whose distance from the $y$-axis is not greater than 6. If $V$ is the intersection of $C_1$ and $C_2$, find the volume of $V$. (Hint: If $T$ is a plane parallel to the $xy$-plane, what does the intersection of $T$ with $V$ look like?)

Being within distance 6 of the $y$ and $x$ axes means that $x^2 + z^2 \leq 36$ and $y^2 + z^2 \leq 36$, so $|x|$ and $|y|$ are both less than or equal to $\sqrt{36 - z^2}$. This means that for any $z \in [-6, 6]$, the possible values of $x$ and $y$ form a square of side $2\sqrt{36 - z^2}$, hence area $144 - 4z^2$. Integrating this from $z = -6$ to $z = 6$ gives 1152.

5. Let $f$ be a 3rd degree polynomial. That is, $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$.
Show that there is at least one number $x_0$ such that $f(x_0) = 0$.

First note that $f(x)$ is continuous, being a polynomial, and that $\lim_{x \to \pm \infty} \frac{f(x)}{x^3} = \lim_{x \to \pm \infty}(a + \frac{b}{x} + \frac{c}{x^2} + \frac{d}{x^3}) = a$.

If $a > 0$ then, when $x$ is sufficiently large and positive, $f(x)$ will be positive, and when $x$ is sufficiently large and negative, $f(x)$ will be negative. Since $f(x)$ is continuous, by the Intermediate Value Theorem, somewhere in between we must have $f(x) = 0$.

If $a < 0$, then for $x$ large and positive we will have $f(x)$ negative, and for $x$ large and negative $f(x)$ will be positive, and we will still have a point in between where $f(x)$ crosses through zero.