

M346 Second Midterm Exam, November 10, 2005

1. Let  $A = \begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix}$ .

- a) Find the eigenvalues and eigenvectors of  $A$ .
- b) Compute  $A^{13421}$ . (No, you do NOT need a calculator for this!)
- c) Compute  $e^A$ .

2. a) In  $\mathbb{R}^3$  with the standard inner product, apply the Gram-Schmidt process to convert the basis  $\{(1, 4, 3)^T, (2, 3, 4)^T, (10, 4, 0)^T\}$  into an orthogonal basis.

b) Find the coordinates of the vector  $\mathbf{v} = (1, -7, 9)^T$  in the orthogonal basis you constructed in part (a).

3. Consider the system of difference equations  $\mathbf{x}(n+1) = A\mathbf{x}(n)$ , where

$$A = \begin{pmatrix} -2 & 0 & 0 \\ -5 & 1 & 2 \\ -3 & 1 & 0 \end{pmatrix} \text{ and } \mathbf{x}(0) = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}.$$

- a) Diagonalize  $A$ .
- b) Find  $\mathbf{x}(n)$  for all  $n$ . (You may express your answer as a linear combination of the eigenvectors of  $A$ , but the coefficients should be explicit.)

4. Consider the nonlinear system of differential equations:

$$\frac{dx_1}{dt} = \ln(x_1 x_2^2) \quad \frac{dx_2}{dt} = \ln(x_1^2 x_2).$$

- a) Find the fixed point (there is only one).
- b) Find a LINEAR system of ODEs that approximates motion near the fixed point.
- c) Find all the stable modes of this linear system. Then find all the unstable modes.

5. a) Find a least-squares solution to  $A\mathbf{x} = \mathbf{b}$ , where  $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$

and  $\mathbf{b} = \begin{pmatrix} 0 \\ 3 \\ 4 \\ 3 \end{pmatrix}$ .

b) Find the equation of the best line through the points  $(-1, 3)^T$ ,  $(0, 1)^T$ ,  $(1, 0)^T$ , and  $(2, -2)^T$ .