1. Let $V = \mathbb{R}_2[t]$ with (standard) basis $B = \{1, t, t^2\}$ and let $W = M_{2,2}$ be the space of 2 by 2 real matrices with (standard) basis $D = \{(1, 0), (0, 1), (0, 0), (0, 1)\}$. Consider the linear transformation $L(p) = \begin{pmatrix} p(1) - p(0) \\ p(2) - p(0) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} p(1) - p(0) \\ p(2) - p(0) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ from $V$ to $W$.

   a) Find the matrix of $L$ relative to the bases $B$ and $D$.
   b) What is the dimension of $\ker(L)$? Find a basis for $\ker(L)$.
   c) What is the dimension of $\operatorname{range}(L)$? Find a basis for $\operatorname{range}(L)$.

2. Let $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 5 & 7 \\ 5 & 10 & 13 & 18 \end{pmatrix}$.

   a) Let $V = \{ \mathbf{x} \in \mathbb{R}^4 | Ax = 0 \}$. What is the dimension of $V$? Find a basis for $V$.
   b) In $\mathbb{R}^3$, consider the vectors $(1, 2, 5)^T$, $(2, 4, 10)^T$, $(3, 5, 13)^T$, and $(4, 7, 18)^T$. Are these vectors linearly independent? Do they span $\mathbb{R}^3$?
   c) Find a basis for the span of the four vectors of part (b).

3. Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be given by $L \left( \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right) = \begin{pmatrix} 8x_1 - 10x_2 \\ 3x_1 - 3x_2 \end{pmatrix}$. On $\mathbb{R}^2$, consider the standard basis $\mathcal{E}$ and the alternate basis $\mathcal{B} = \{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix} \}$. Finally, let $v = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$.

   a) Find $P_{\mathcal{EB}}$, $P_{\mathcal{BE}}$, $[v]_\mathcal{E}$ and $[v]_\mathcal{B}$.
   b) Find the matrix $[L]_\mathcal{E}$ and the matrix $[L]_\mathcal{B}$.

4. The two parts of this problem are NOT connected.

   a) In $\mathbb{R}_2[t]$, consider the vectors $\mathbf{b}_1 = 1 + t + 2t^2$, $\mathbf{b}_2 = 2 + 3t + 5t^2$ and $\mathbf{b}_3 = 3 + 7t + 9t^2$. Do these vectors form a basis for $\mathbb{R}_2[t]$? If so, find $[v]_\mathcal{B}$, where $v = 1 - 2t$. If not, find constants $a_1, a_2, a_3$, not all zero, such that $a_1 \mathbf{b}_1 + a_2 \mathbf{b}_2 + a_3 \mathbf{b}_3 = 0$.
   b) In $\mathbb{R}_3[t]$, let $V$ be the set of polynomials $\mathbf{p}$ for which $\mathbf{p}(0) = \mathbf{p}(1) = 0$. Find a basis for $V$. 

1
5. True of False? Each question is worth 4 points. You do NOT need to justify your answers, and partial credit will NOT be given.

a) The plane $x_1 + 3x_2 - 4x_3 = 0$ is a subspace of $\mathbb{R}^3$.

b) If $A$ is a $3 \times 5$ matrix, then the dimension of the null space of $A$ is at least 2.

c) Let $L : \mathbb{R}_5[t] \to \mathbb{R}^3$ be a linear transformation. If $L$ is onto, then the kernel of $L$ is 2-dimensional.

d) Let $B = \{b_1, \ldots, b_n\}$ be a basis for a vector space $V$. If $n$ vectors $d_1, \ldots, d_n$ span $V$, then the vectors $[d_1]_B, \ldots, [d_n]_B$ are linearly independent.

e) Every linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^4$ is multiplication by a $5 \times 4$ matrix.