M362K Third Midterm Exam. April 16, 2004

Problem 1. Manipulating continuous variables

Let X be a continuous random variable with pdf

$$f_X(x) = \begin{cases} 6x^{-7} & \text{if } x \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

a) Compute the expectation $E(X)$

b) Compute the variance $Var(X)$

c) Let $Y = X^2$. Compute the pdf $f_Y(y)$.

Problem 2. Continuous joint distributions

Let X and Y be continuous random variables with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} xe^{-(x+y)} & \text{if } x > 0 \text{ and } y > 0, \\ 0 & \text{otherwise} \end{cases}$$

a) Are X and Y independent random variables? Why or why not?

b) Let $Z = X + Y$. Find the cdf $f_Z(z)$ for all values of z.

Problem 3. A dicey problem Two fair dice are rolled. Let X be the value of the higher die, and let Y be the value of the lower die. (If the two dice give the same value, say double 4's, then both X and Y would equal 4, while if we got a 5 and a 3 we would have $X = 5$ and $Y = 3$).

a) Find the joint pdf $f_{X,Y}(x,y)$ for all posible pairs (x,y).

b) Compute the marginal pdf's $f_X(x)$ and $f_Y(y)$.

c) Find $F_X(3)$.

Problem 4. Lottery tickets A lottery is designed so that each ticket has a 10% change of paying \$ 2, a 4% chance of paying \$ 5, a 1% chance of paying \$ 10, and an 85% chance of paying nothing. You buy a ticket, and call its value X.

a) What is the expectation $E(X)$?

b) Compute the variance $Var(X)$ and the standard deviation σ_x.

c) Suppose you buy 100 lottery tickets, where each ticket is independent of the others. Let Y be the total value of all 100 tickets put together. Compute $E(Y)$ and σ_y.