$$x^2 + y^2 = 25$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Downarrow$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

2 Big applications

1) Equations where you cant write $y = f(x)$.

2) Inverse functions.

$$y = \ln(x) \Leftrightarrow x = e^y$$

$$1 = e^y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$$y = \tan^{-1}(x) \qquad x = \tan(y)$$

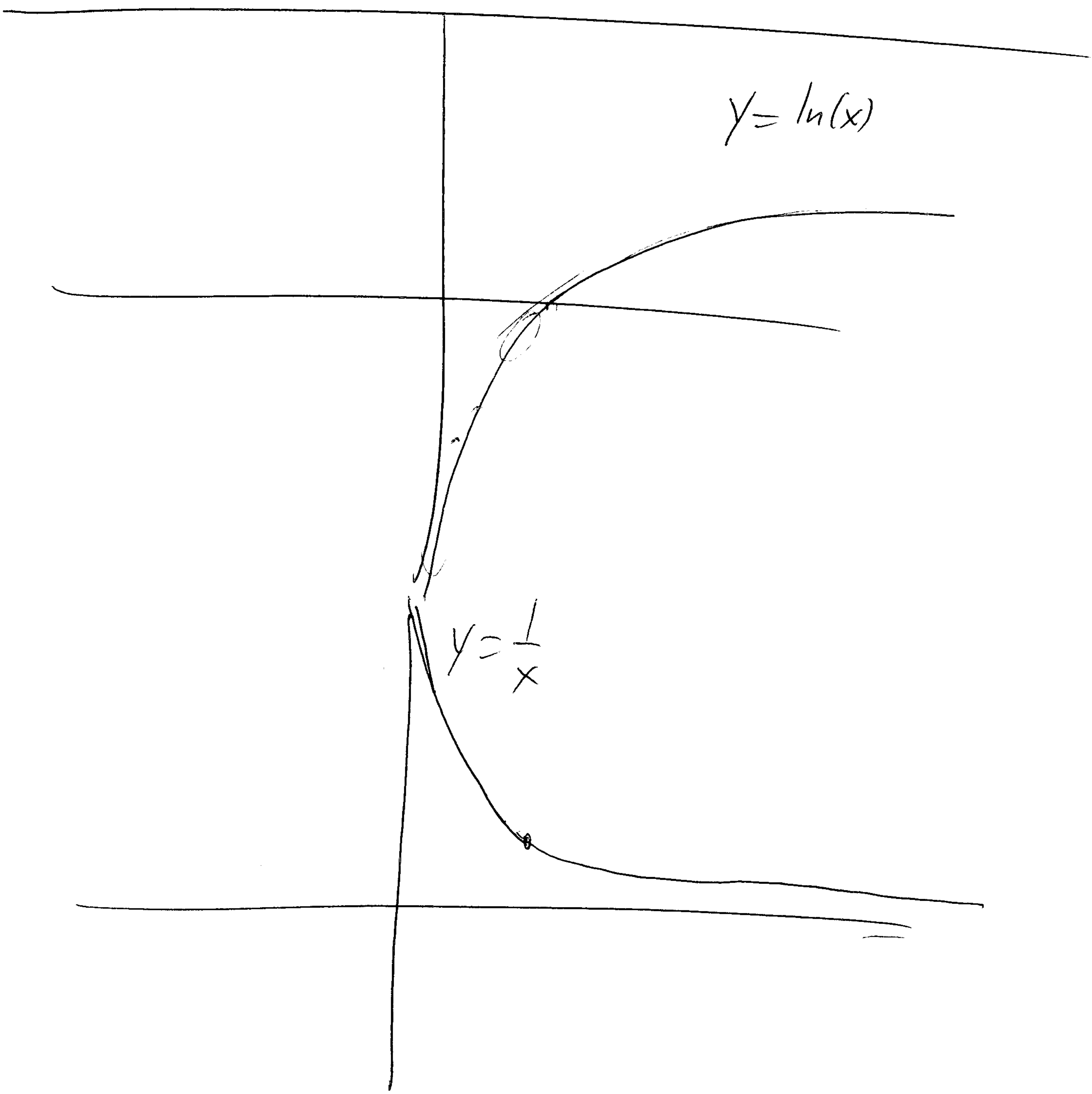
$$1 = \sec^2(y)\frac{dy}{dx}$$

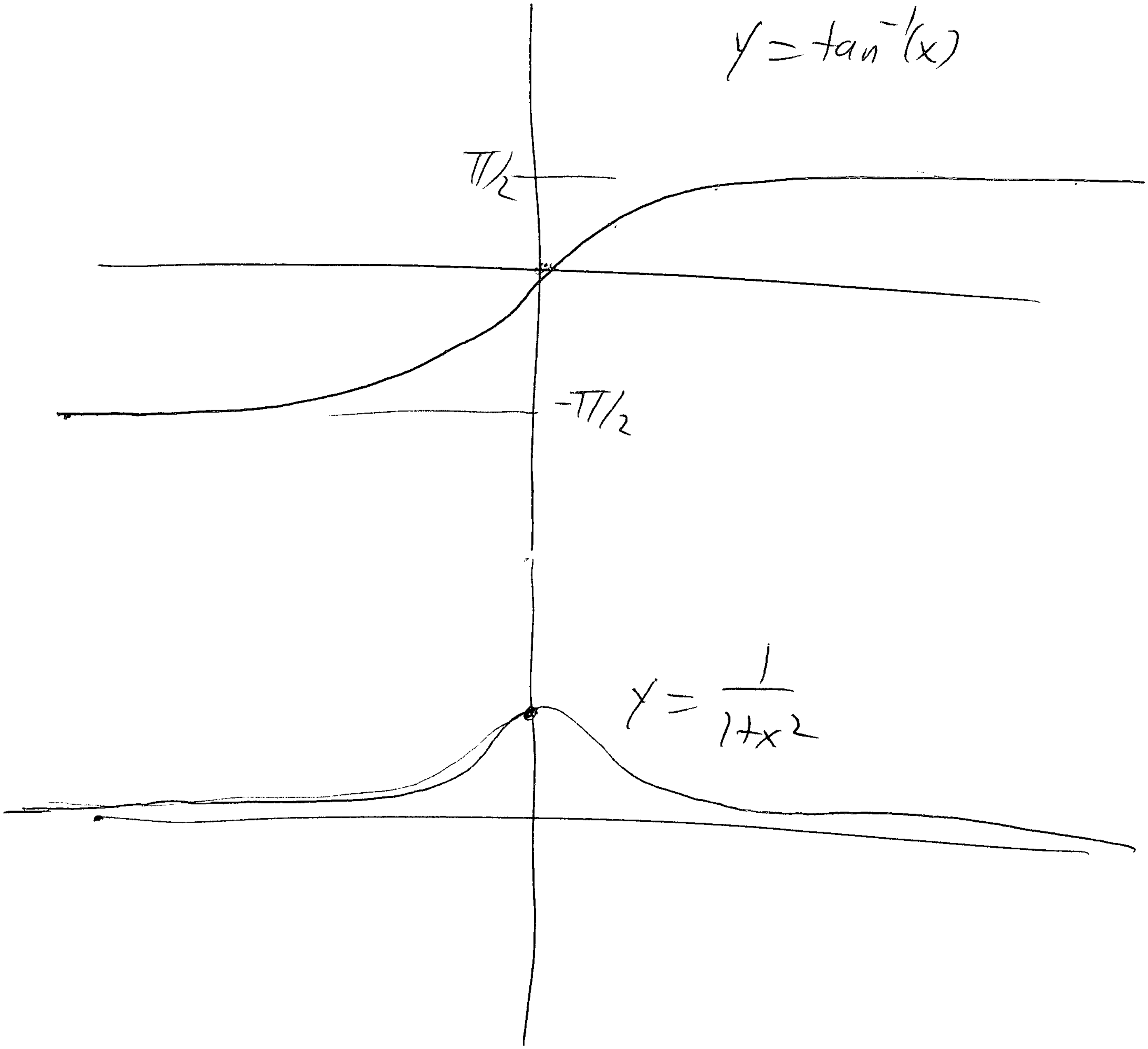
$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} = \frac{1}{1+\tan^2(y)} = \frac{1}{1+x^2}$$

$y = \sin^{-1}(x)$; $x = \sin(y)$

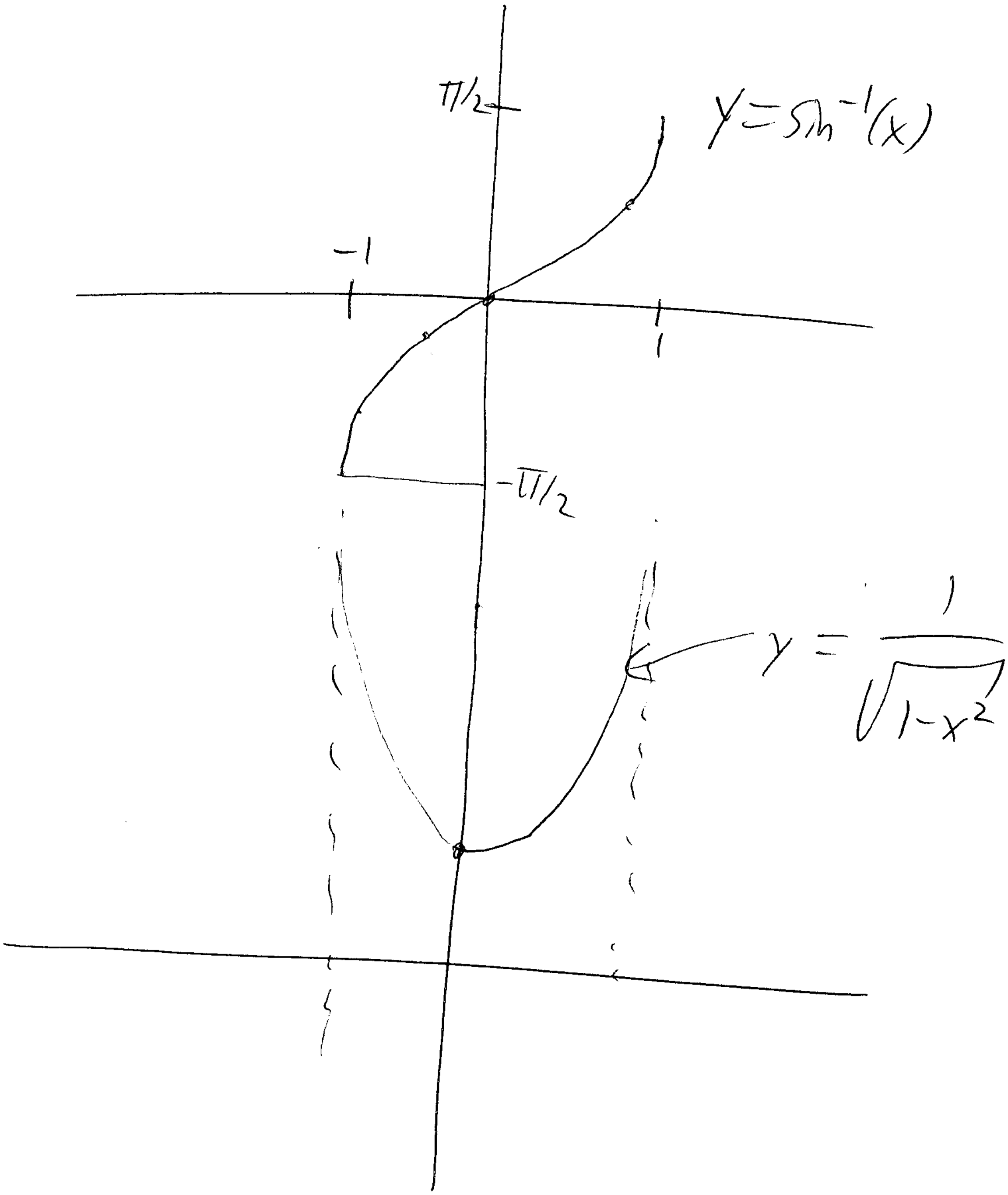
$$1 = \cos(y)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-\sin^2(y)}} = \frac{1}{\sqrt{1-x^2}}$$





$y = \tan^{-1}(x)$
$\pi/2$
$-\pi/2$
$y = \frac{1}{1+x^2}$


$\pi/2$
$y = \sin^{-1}(x)$
-1
1
$-\pi/2$
$y = \frac{1}{\sqrt{1-x^2}}$

$$\frac{d(\ln y)}{dx} = \frac{1}{y} \cdot \frac{dy}{dx}$$

Logarithmic derivative

$$\frac{dy}{dx} = y \cdot \frac{d \ln(y)}{dx}$$

$$y = \frac{x^{3/4} (x^2+1)^{1/2}}{(3x+2)^5}$$

Compute $\frac{dy}{dx}$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^r) = r \ln(a)$$

$$\ln(y) = \ln(x^{3/4}) + \ln\left((x^2+1)^{1/2}\right) - \ln\left((3x+2)^5\right)$$

$$= \frac{3}{4}\ln(x) + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2)$$

$$\frac{d}{dx}\ln(y) = \frac{3}{4x} + \frac{\cancel{1}}{\cancel{2}} \frac{\cancel{2}x}{(x^2+1)} - 5\frac{3}{3x+2}$$

$$= \left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$$

$$\frac{dy}{dx} = y \cdot \left(\quad '' \quad \right) = \left(\frac{x^{3/4}(x^2+1)^{1/2}}{(3x+2)^5}\right)\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$$

$$y = x^x$$

$$\ln y = \ln(x^x) = \overset{u}{x}\ \overset{v}{\ln(x)}$$

$$\qquad\quad a^r \qquad\quad r\ \ln(a)$$

$$\frac{d}{dx}\ln(y) = x \cdot \frac{1}{x} + 1 \cdot \ln(x) = 1 + \ln(x)$$

$$u\ \frac{dv}{dx} + \frac{du}{dx}\ v$$

$$\frac{d}{dx}y = y\ \frac{d}{dx}\ln(y) = y\,(1+\ln(x)) = x^x(1+\ln(x))$$

When not to use it:

$y = \sin(x)$

$$\ln(y) = \ln(\sin(x))$$

$$\frac{d\ln(y)}{dx} = \frac{1}{\sin(x)} \cdot \cos(x) = \frac{dy/dx}{y}$$

$$\frac{dy}{dx} = y\ \frac{d\ln(y)}{dx} = \frac{\sin(x) \cdot \cos(x)}{\sin(x)} = \cos(x)$$

USE IT:

$$y = a^x$$

$$\ln(y) = x \ln(a)$$

$$\frac{d \ln(y)}{dx} = \ln(a)$$

$$\frac{dy}{dx} = y \ln(a) = a^x \ln(a)$$

$$y = x^n$$

$$\ln y = n \ln x$$

$$\frac{d \ln y}{dx} = \frac{n}{x}$$

$$\frac{dy}{dx} = y \cdot \frac{n}{x} = \frac{x^n \cdot n}{x} = n x^{n-1}$$

Why not other bases.

$$y = \log_{10}(x) \qquad x = 10^y$$

$$1 = 10^y \ln(10) \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{10^y \ln(10)} = \frac{1}{x \ln(10)}$$

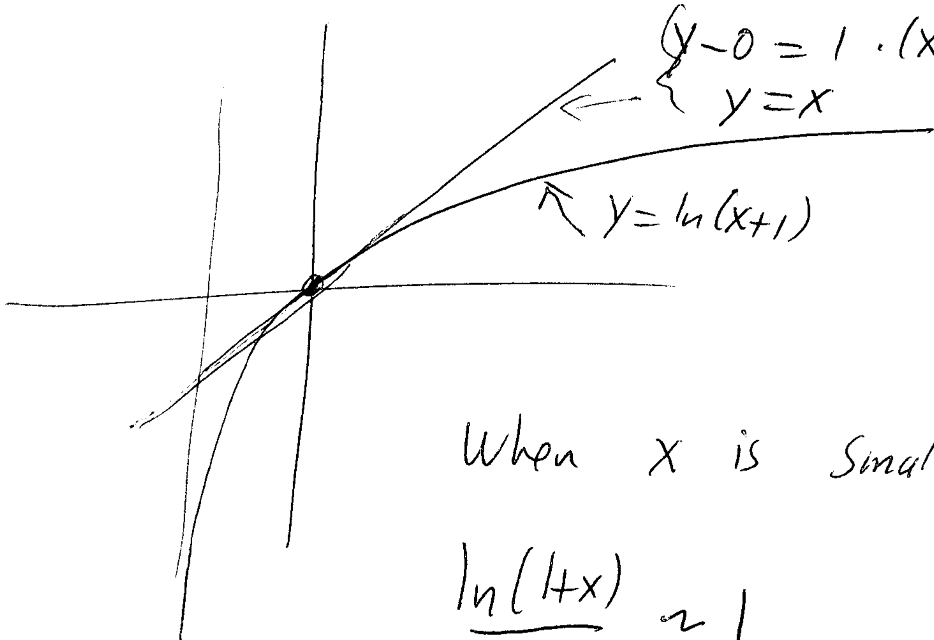
$$\underline{\log_{10}(x)} = \frac{\log_e(x)}{\cancel{\text{Log}_{[illegible]}([illegible])}\ \log_e(10)}$$

$$\frac{d}{dx}\log_{10}(x) = \frac{1/x}{\ln(10)} = \frac{1}{x\ln(10)}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$f(x) = \ln(1+x)$ $\qquad$ $f(0) = 0$

$f'(x) = \dfrac{1}{1+x}$ $\qquad$ $f'(0) = 1$



When x is small, $\ln(1+x) \approx x$

$$\frac{\ln(1+x)}{x} \approx 1$$

$$e^{\left(\frac{\ln(1+x)}{x}\right)} \approx e$$

$x = \frac{1}{n}$

$$(1+x)^{1/x} \approx e$$

$$\boxed{e = \lim_{x\to 0}(1+x)^{1/x} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.718281828\ldots}$$

$$\boxed{\text{Claim: } e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n}$$

$$\ln\left(\left(1+\frac{x}{n}\right)^n\right) = n \ln\left(1+\frac{x}{n}\right)$$

$$\approx n\left(\frac{x}{n}\right) = x$$

If $f(t)$ = position at time t.

$$f'(t) = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h}$$

$$= \lim_{h \to 0} \frac{\text{distance}}{\text{time}} = \text{velocity at time } t.$$

$$f''(t) = \lim_{h \to 0} \frac{f'(t+h) - f'(t)}{h} = \lim \frac{\Delta \text{ velocity}}{\Delta \text{ time}}$$

$= \text{acceleration}.$

Force = mass · acceleration.

$$\text{acceleration} = \frac{\text{Force}}{\text{mass}}.$$

For gravity, $a(t) = -32 \text{ ft}/\text{sec}^2 = -9.8 \text{ m}/\text{s}^2$

$$V(t) = V_0 - 32t$$

$$X(t) = V_0 t - 16t^2 + X_0$$

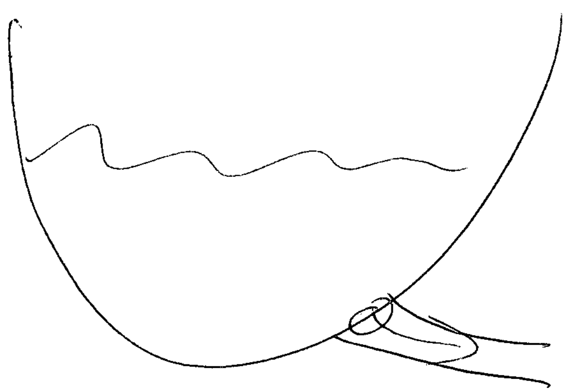
$$\frac{d^2x}{dt^2} = -32$$

$$F = -kx$$

$$a = -\frac{k}{m}x$$

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}\,x(t)$$

differential equation



$V(t)$ = volume

$V'(t)$ = rate of volume changing

Leakage rate = $-V'(t)$