position $= f(t)$

velocity $= f'(t)$

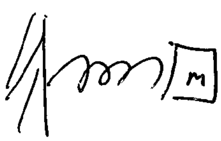
acceleration $= f''(t)$

acceleration $= \dfrac{\text{Force}}{\text{mass}}$

Falling object ; $a(t) = -32\ ft/sec^2 = f''(t)$

$$V(t) = V_0 - 32t = f'(t)$$

$$X(t) = X_0 + V_0 t - 16t^2 = f(t)$$



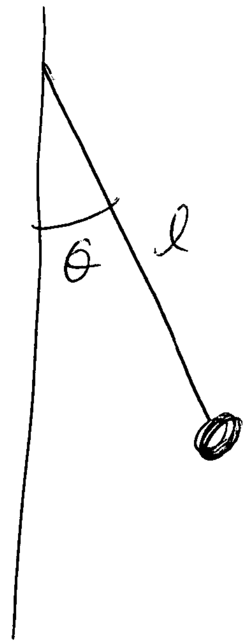
$$F = -kx$$

$$a(t) = -\frac{k}{m} x(t)$$

$$f''(t) = -\frac{k}{m} f(t)$$

Suppose $k = m = 1$,

$$f''(t) = -f(t)$$

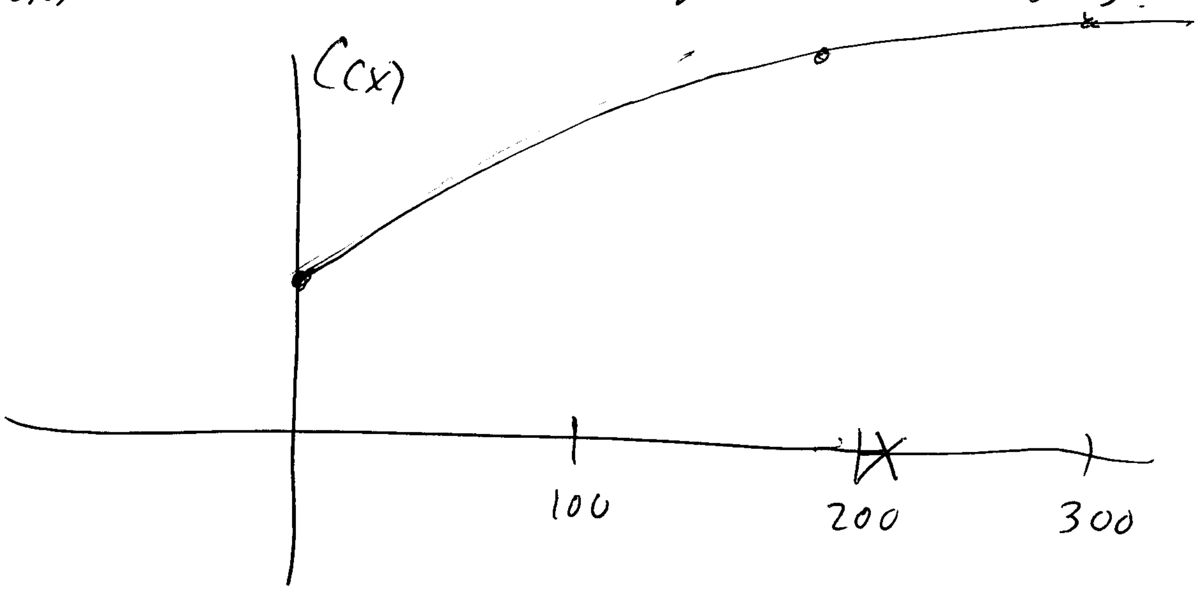


$$\theta = f(t)$$

$$f''(t) = -\frac{g}{\ell}\sin(f(t))$$

$$\approx -\frac{g}{\ell} f(t)$$

$C(x) =$ cost of making x widgets.



$$\text{Marginal cost} = C'(x) = \lim_{h \to 0} \frac{C(x+h) - C(x)}{h}$$

$$\approx C(x+1) - C(x)$$

$=$ cost of one more widget.

$R(x) =$ revenue from x widgets

$P(x) = R(x) - C(x)$ = profit from x widgets.

marginal profit $= P'(x)$

Population growth.

$P(t) =$ population at time t.

$P'(t) =$ rate of population growth.

$=$ birth rate − death rate + net immigration.

$=$ (~~percent~~ relative growth rate) $\cdot P$

relative growth rate $= \dfrac{P'(t)}{P(t)} =$ Logarithmic derivative.

$$\boxed{f'(t) = r\, f(t)}$$

$$f(t) = f(0)\, e^{rt}$$

\$1,000 at 6% interest, compounded yearly.

$M(0) = 1000$

$M(1) = 1000 + .06(1000) = (1.06)\,1000$

$M(2) = M(1) + .06\,M(1) = (1.06)M(1) = (1.06)^2 \cdot 1000$

$M(n) = (1.06)^n \cdot (1000)$

Compounded 2x/year.

$M(n) = (1.03)^{2n} \cdot (1000)$

Compounded 12x/yr.

$M(n) = \left(1 + \frac{.06}{12}\right)^{12n} (1000)$

Compounded daily

$M(n) = \left(1 + \frac{.06}{365}\right)^{365n} (1000)$

Compounded N times/yr.

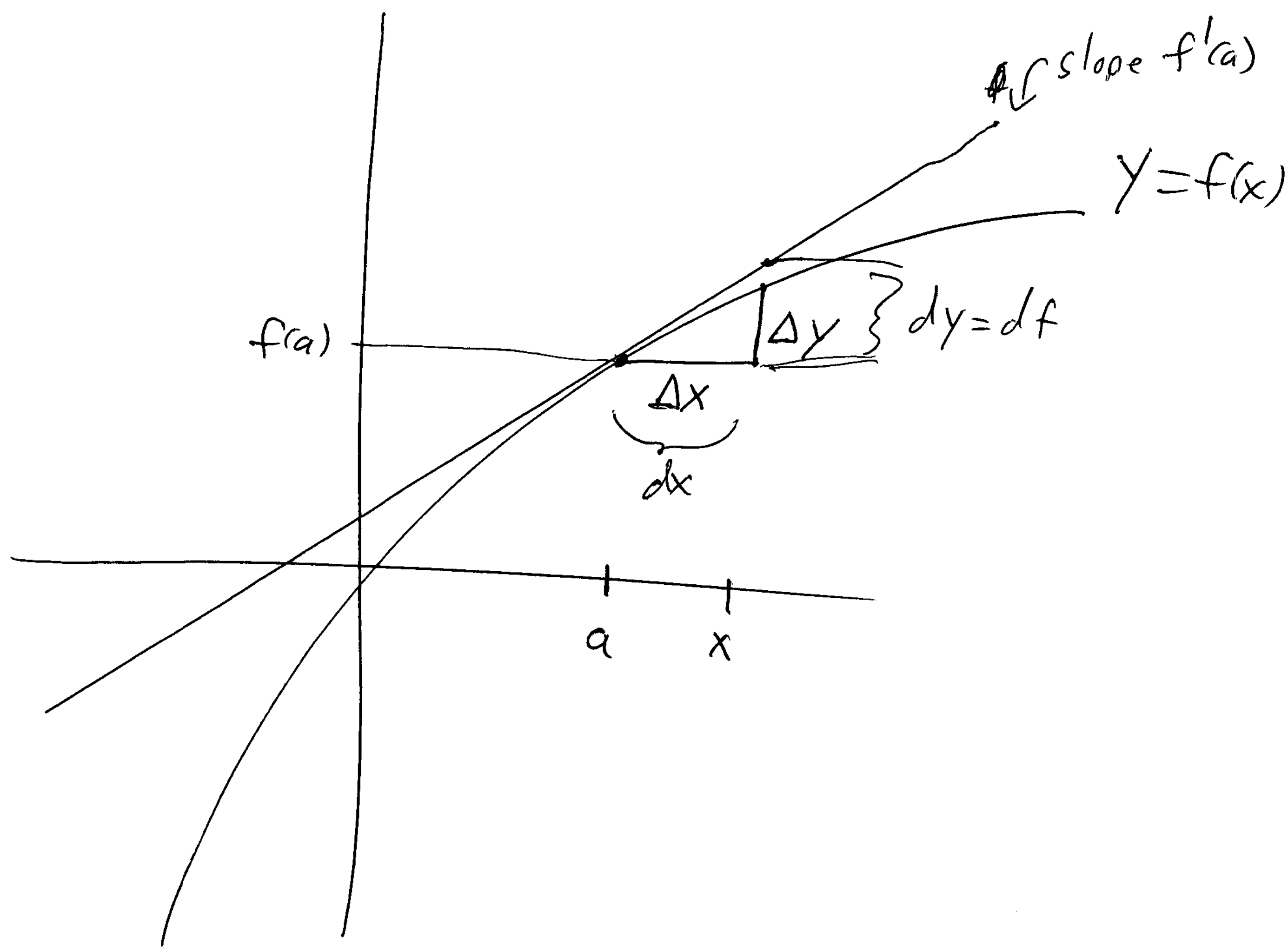
$$\left(1+\frac{.06}{N}\right)^{Nn} \cdot 1000$$

$$\lim_{N\to\infty}\left(1+\frac{.06}{N}\right)^{Nn}\cdot 1000$$

$$=\left(\lim_{N\to\infty}\left(1.+\frac{.06}{N}\right)^{N}\right)^{n}\cdot 1000$$

$$\boxed{\lim_{N\to\infty}\left(1+\frac{X}{N}\right)^{N}=e^{X}} \quad = \cancel{e}\left(e^{.06}\right)^{n}\cdot 1000$$

$$= e^{\overset{r}{.06}\,\overset{t}{n}}\cdot 1000$$



Whenever $x \approx a$, $\dfrac{f(x)-f(a)}{x-a} \approx f'(a)$

Δy

$$f(x)-f(a) \approx f'(a)(x-a) = f'(a)\,\Delta x$$

$$\boxed{f(x) \approx f'(a)(x-a)+f(a)}$$

$\Delta x =$ change in $x = x-a$

$\Delta y = \Delta f =$ change in $f(x) = f(x)-f(a)$

$dx =$ change in $x = \Delta x$

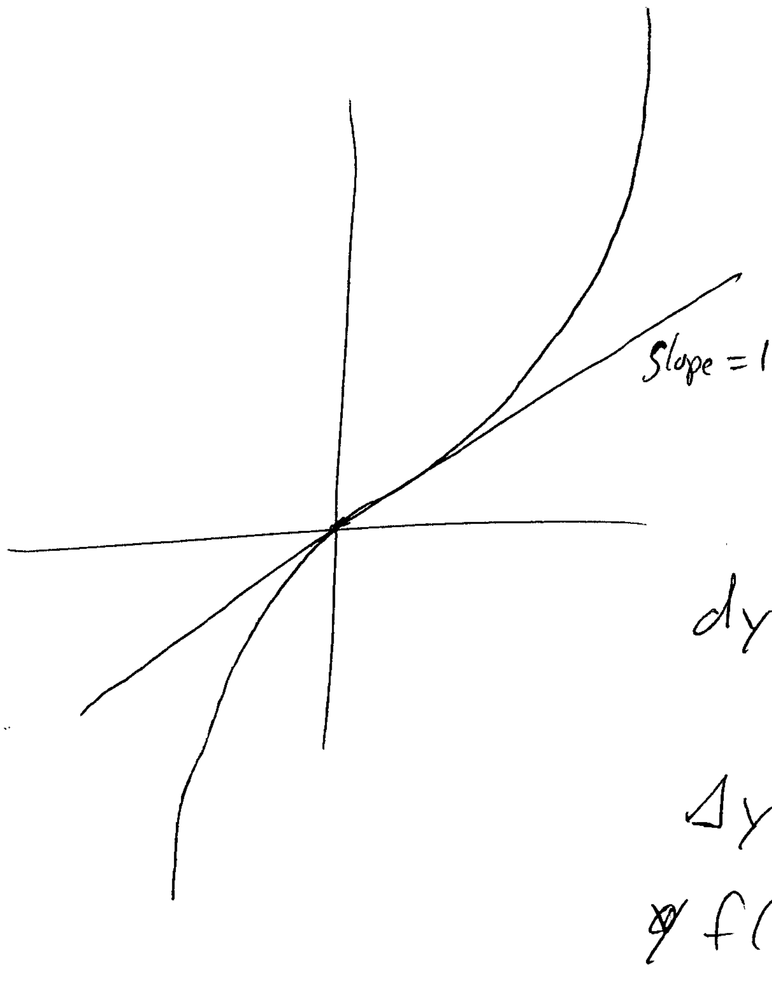
$dy = df =$ what the change would be if you tracked the tangent line

$$dy = f'(a)\,dx \qquad f'(a) = \frac{dy}{dx}$$

$\Delta y \approx dy$ as long as dx is small.

$\tan(.02 \text{ radians})$

$$\left.\begin{aligned} x &= .02 \\ a &= 0 \end{aligned}\right\} dx = .02$$



$f(x) = \tan(x)$

$f'(x) = \sec^2(x) = \frac{1}{\cos^2(x)}$

$f(0) = 0$

$f'(0) = 1$

$dy = 1 \cdot dx$

$\quad = 1 \cdot (.02)$

$\Delta y \approx .02$

$f(.02) \approx f(0) + .02$

$\quad = 0 + .02 = .02$

$f(x) = e^{x^2-1}$

$f(1.01)$

$f'(x) = 2x\, e^{x^2-1}$

$x = 1.01$

$a = 1$

$a = 1$

$dx = .01$

$f(a) = e^{1^2-1} = e^0 = 1$

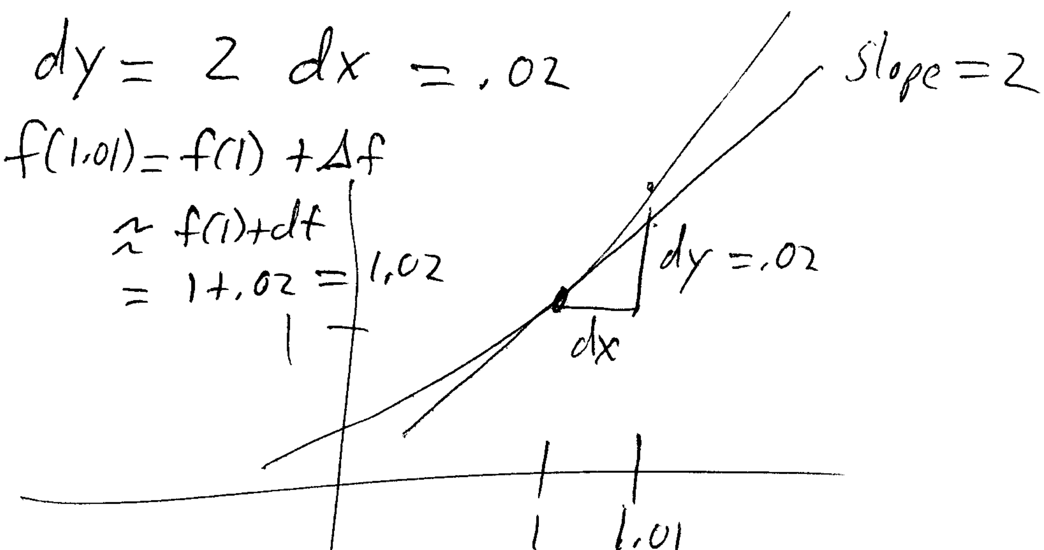
$f'(a) = 2 \cdot 1 \cdot e^0 = 2$

$dy = 2\, dx = .02$

$f(1.01) = f(1) + \Delta f$

$\approx f(1) + df$

$= 1 + .02 = 1.02$



Rise = slope · run

$dy = f'(a)\, dx$

$$\ln(\tan(46^\circ)) = \ln(\tan(45^\circ + 1^\circ))$$

$$= \ln\left(\tan\left(\frac{\pi}{4} + \frac{\pi}{180}\right)\right)$$

$$f(x) = \ln(\tan(x))$$

$$a = \frac{\pi}{4} \qquad dx = \frac{\pi}{180}$$

$$f(a) = \ln\left(\tan\left(\frac{\pi}{4}\right)\right)$$

$$= \ln(1) = 0$$

$$f'(x) = \frac{\frac{d}{dx}\tan(x)}{\tan(x)} = \frac{\sec^2(x)}{\tan(x)} \quad \cancel{\frac{2}{1} = 2}$$

$$f'(a) = 2$$

$$dy = 2\,dx$$

$$= \frac{\pi}{90} \approx \frac{3.14}{90} \approx .035$$

$$f\left(\frac{\pi}{4} + \frac{\pi}{180}\right) \approx f\left(\frac{\pi}{4}\right) + dy = 0 + \frac{\pi}{90} = \frac{\pi}{90}$$

$(1.03)^{2.3}$

$f(x) = x^{2.3}$ $\quad a = 1$

$f'(x) = 2.3x^{1.3}$ $\quad x = 1.03$

$dx = .03$

$f(a) = 1$

$f'(a) = 2.3$

$dy = 2.3\,dx$

$= (2.3)(.03)$

$= .069$