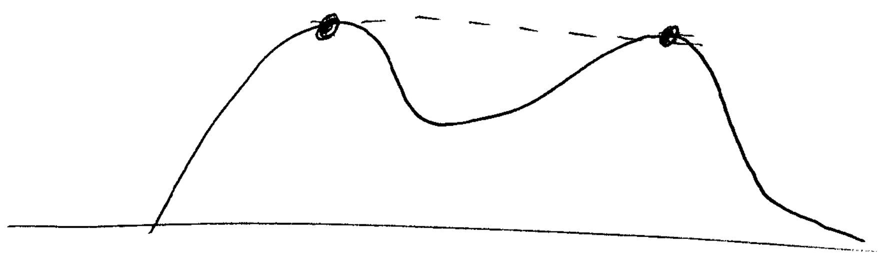
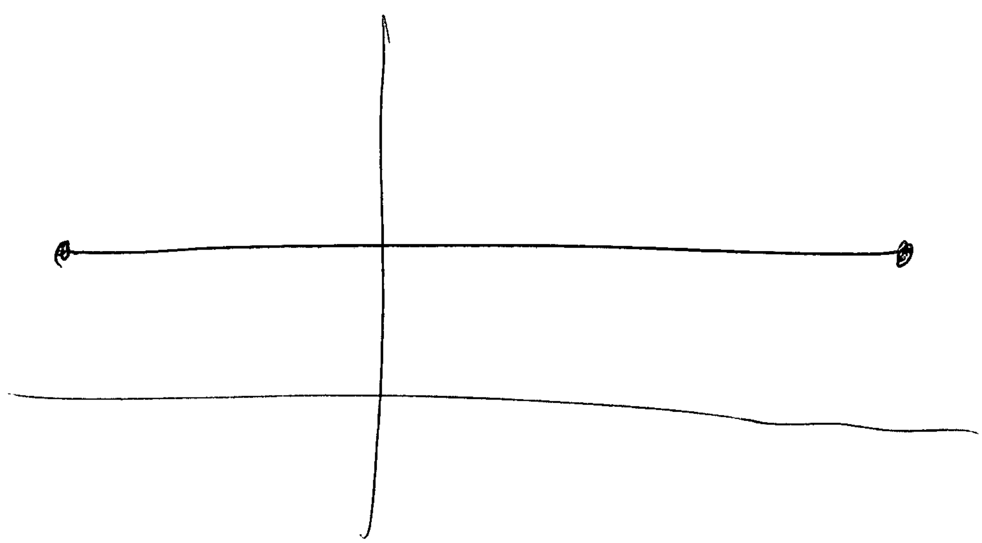
Critical #s = Critical pts.

global max = absolute max

If $f(x)$ is continuous on $[a,b]$, then $f(x)$ has a global ~~max/~~ (min) on $[a,b]$.





If f is cont and has a local maximum at $x=a$*, what is $f'(a)$?

* and is defined on both sides of a.

$f'(a)$ is not > 0.

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to a^+} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a^+} \frac{(\le 0)}{>0}$$

$$= \le 0$$

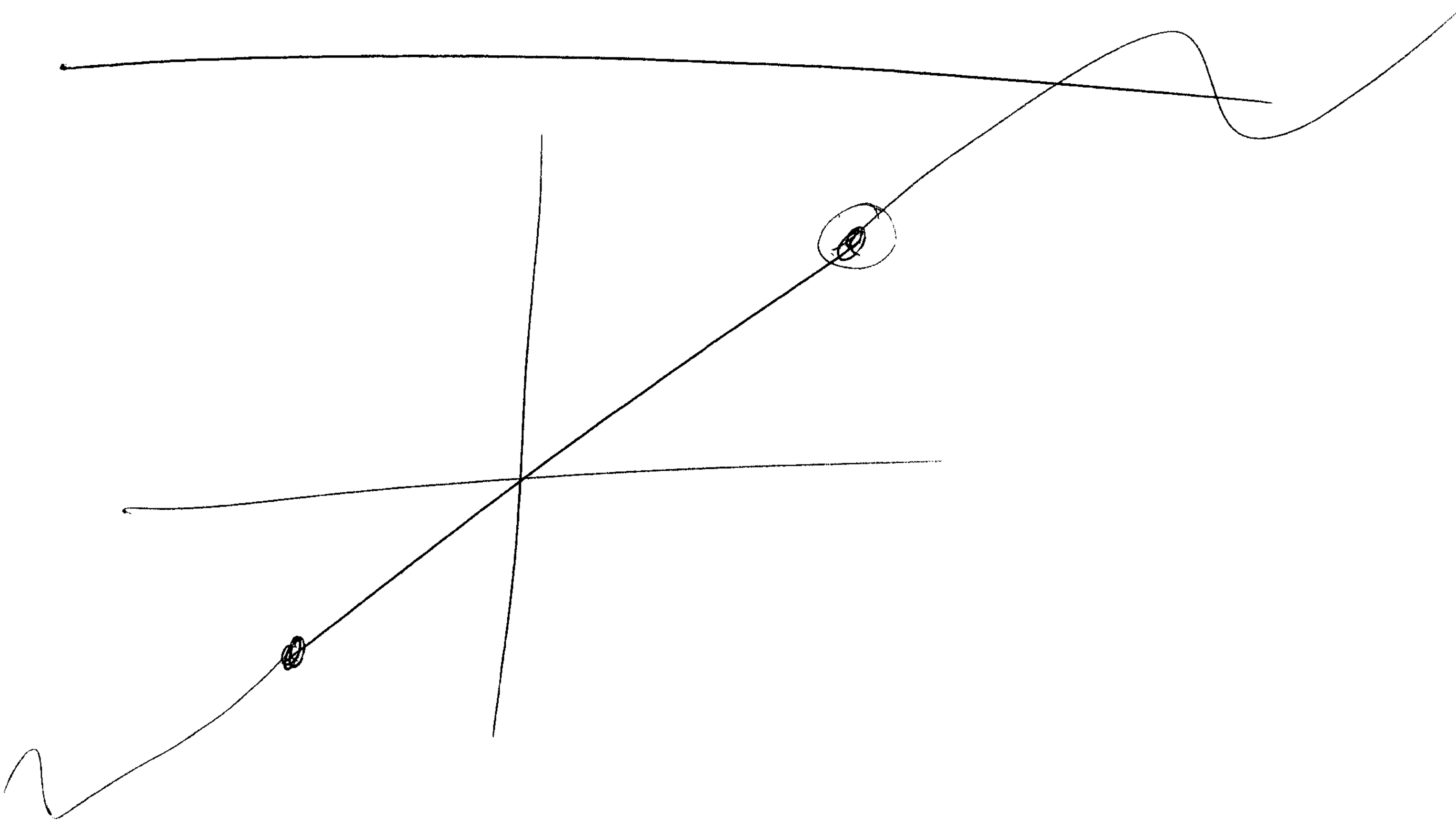
$f'(a)$ is not < 0

$$f'(a) = \lim_{x \to a^-} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a^-} \frac{(\le 0)}{<0} \ge 0$$

Fermat's Thm

If $f(x)$ ~~has a local ma~~ is continuous and has a local max at $x=a$ and is defined on both sides of $x=a$, then either

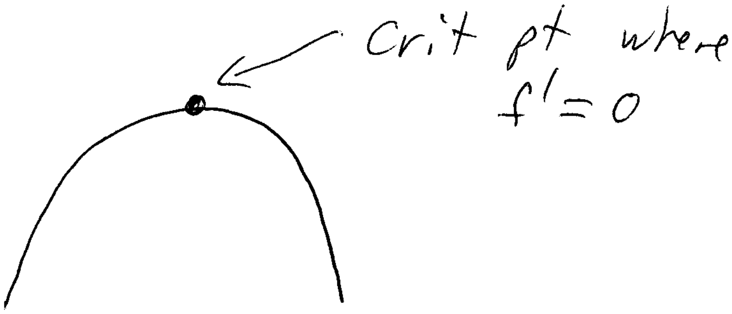
1) $f'(a)=0$, or
2) $f'(a)$ DNE.

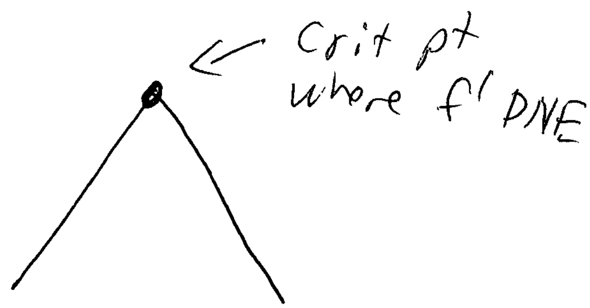


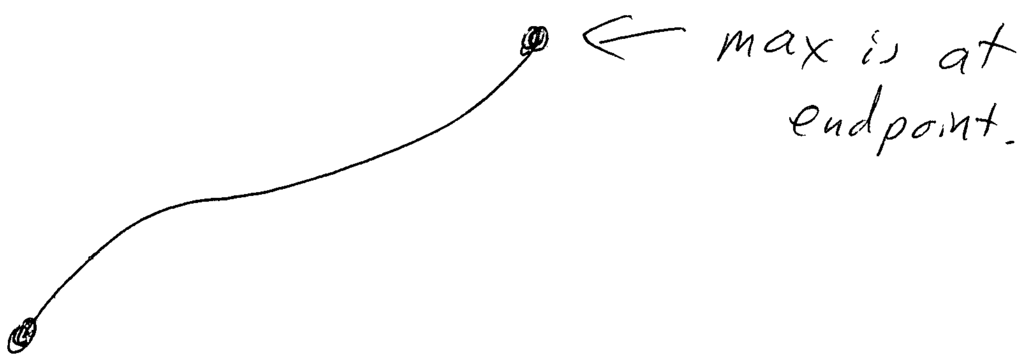
A critical point (critical #) is a value of x where either

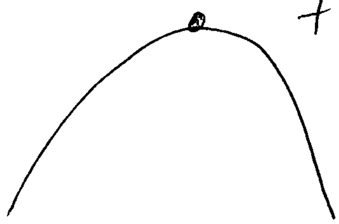
1) $f'(x) = 0$, or
2) $f'(x)$ DNE.

The maximum of a function on $[a,b]$ is either at ~~at~~ a crit. pt. or at an endpt.

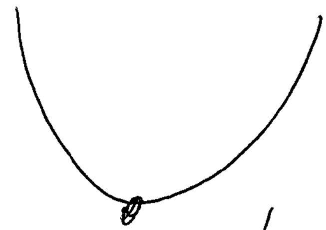




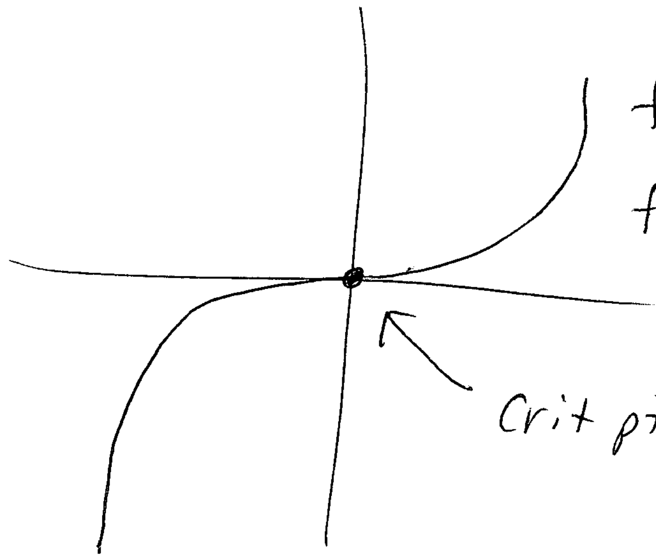




$f'=0$, max



$f'=0$, min

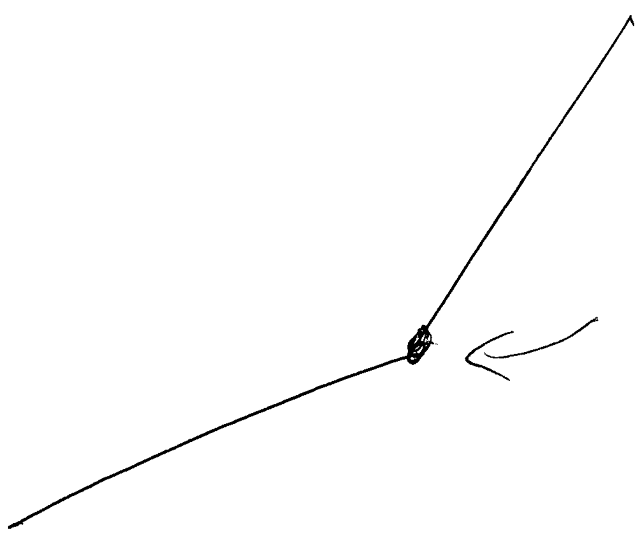


$f(x)=x^3$

$f'(x)=3x^2$

$f'(0)=0$

crit pt, not max, not min



crit pt, not max,
not min

Closed interval method:

To maximize $f(x)$ on $[a,b]$,

1) Find critical pts on (a,b)
2) Evaluate $f(x)$ at crit pts & endpoints.
3) Pick the biggest.

Find the max/min of $f(x) = x^3 - 3x$ on $[-3, 2]$

$f'(x) = 3x^2 - 3$ (exists everywhere)

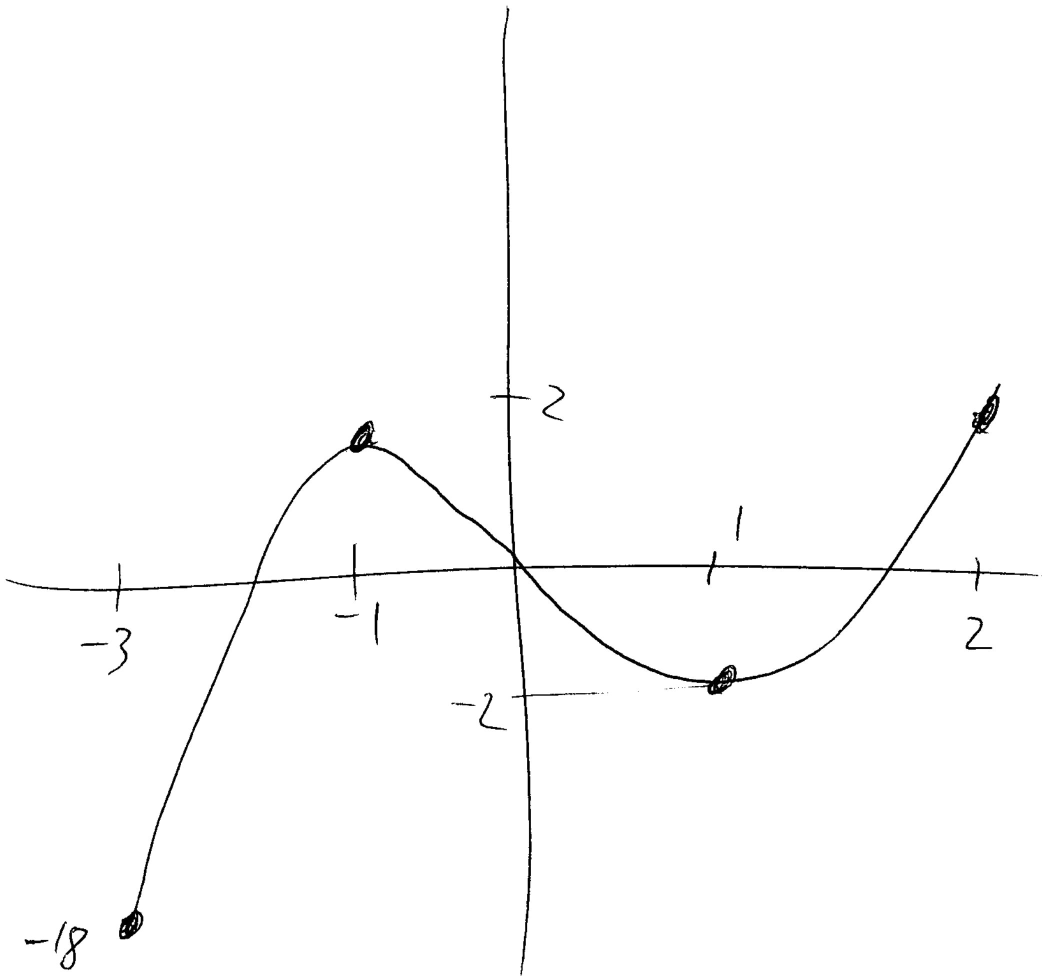
$0 = f'(x) = 3x^2 - 3$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

x	$f(x)$	
-3	-18	min
-1	2	
1	-2	max
2	2	


2
1
-3
-1
2
-2
-18

Find max/min of $f(x) = 6x - x^2$

on $[-1, 2]$

$= 9 - (x-3)^2$

can do by algebra

(complete the square)

$f'(x) = 6 - 2x$ No crit pts.

x	$f(x)$	
-1	-7	min
2	8	max

---

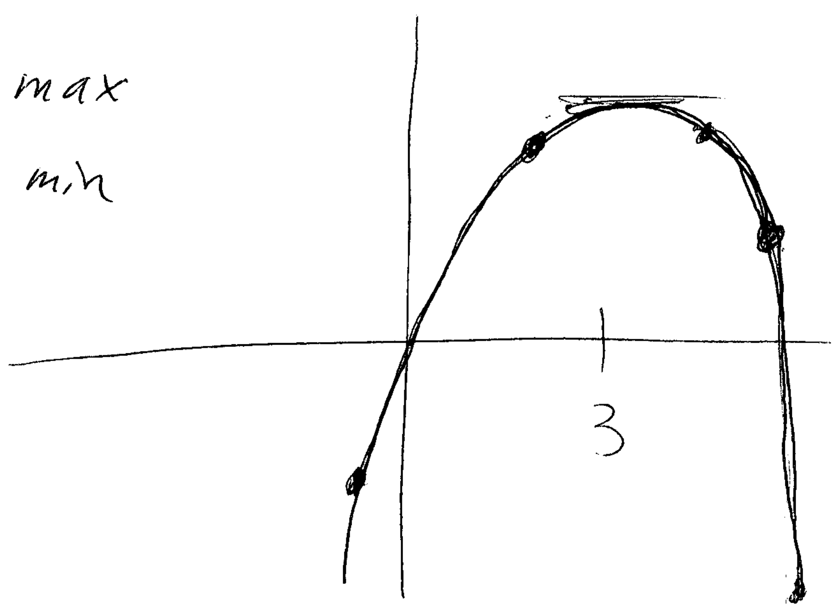
on $[4, 7]$ no crit pts.

x	$f(x)$	
4	8	max
7	-7	min

---

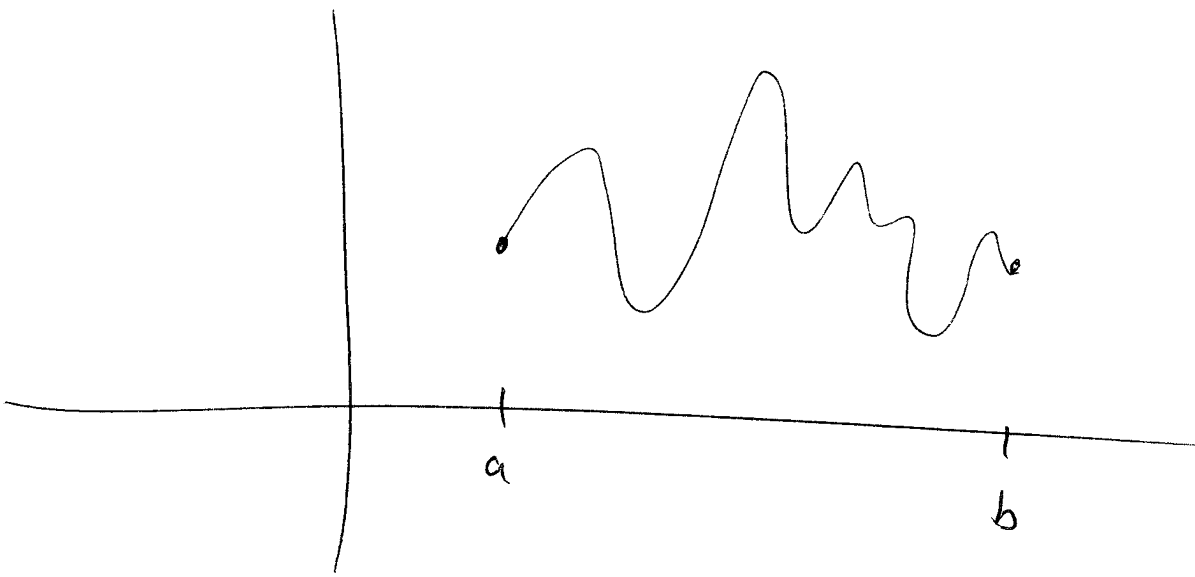
on $[2, 5]$ Crit pt at $x = 3$

x	$f(x)$	
2	8	
3	9	$\leftarrow$ max
5	5	$\leftarrow$ min



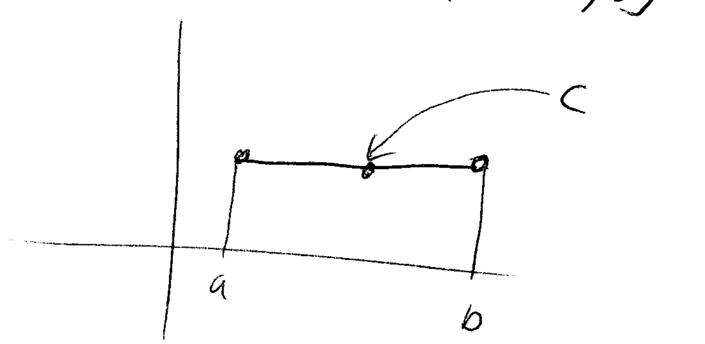
Rolle's Thm

If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), and if $f(a)=f(b)$, then there is at least one pt $c \in (a,b)$ where $f'(c)=0$.



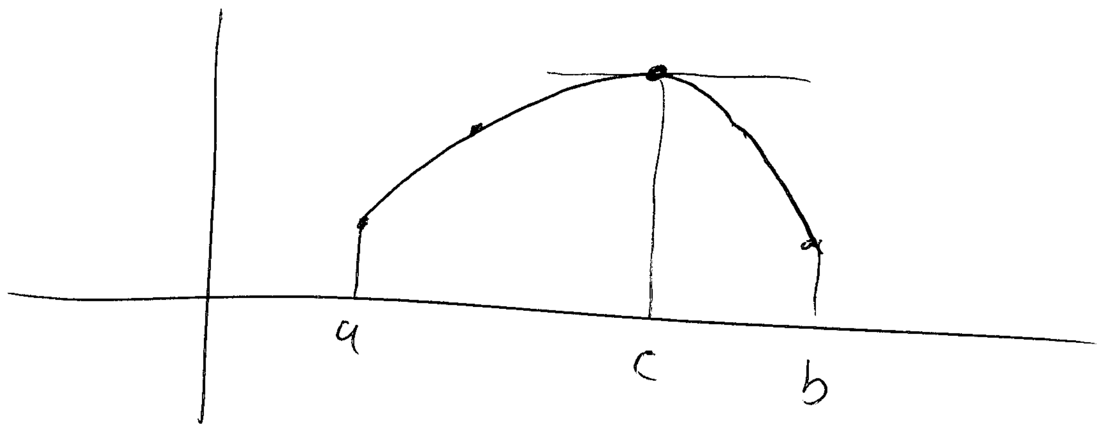
~~Bd~~ Pf: 3 cases:

i) $f(x)$ is constant on $[a,b]$

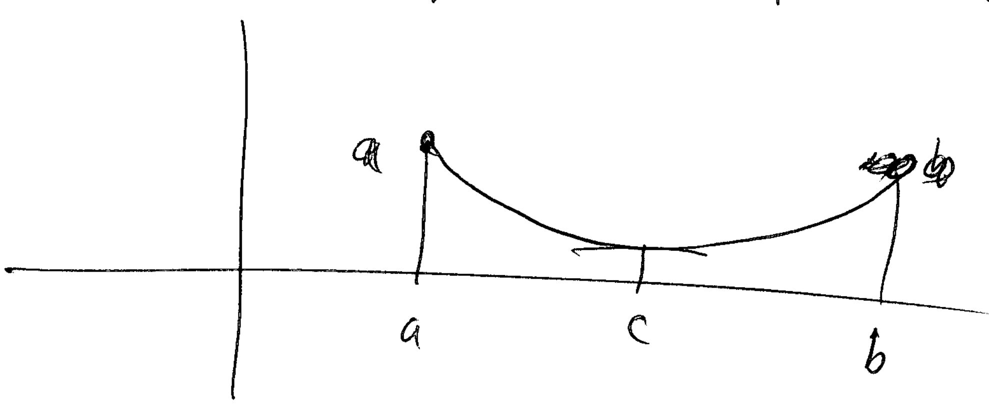


ii) There is a point in the middle with $f(x) > f(a)$.

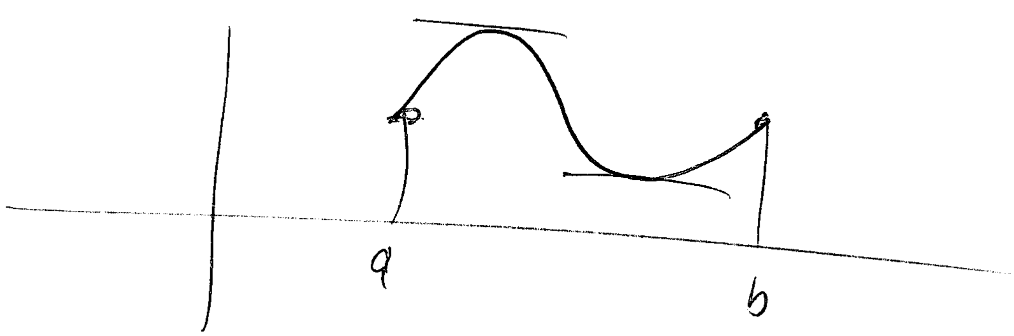
Then a_0 is not global max, nor is b.
But global max exists, and $f' = 0$ at global max.



iii) There is a pt with $f(x) < f(a)$

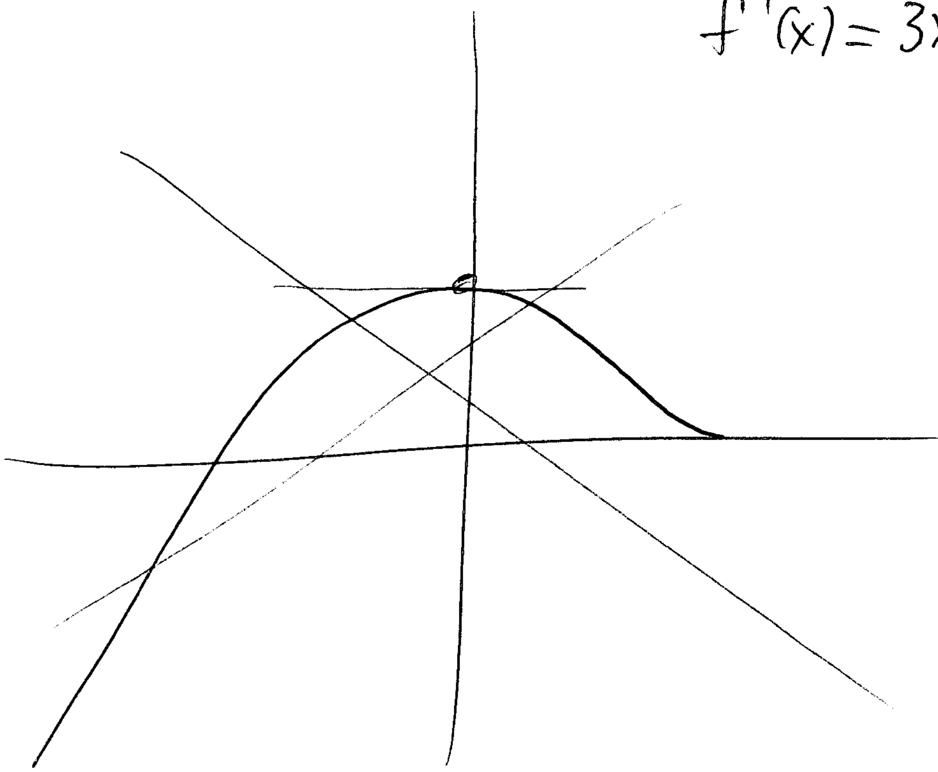


Cases ii) and iii) overlap



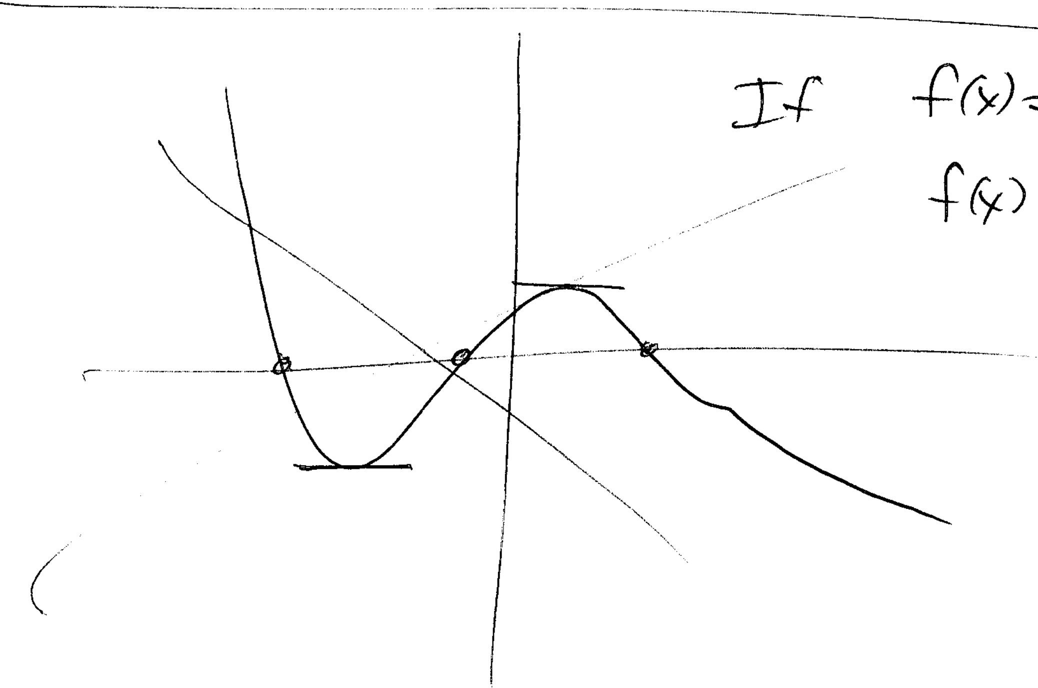
Claim! $f(x) = x^3 + x - 1$ has exactly one real root

$f'(x) = 3x^2 + 1 > 0$



Thm The polynomial $f(x) = x^n + \cdots$ has at most n roots.

True for $n=1$.



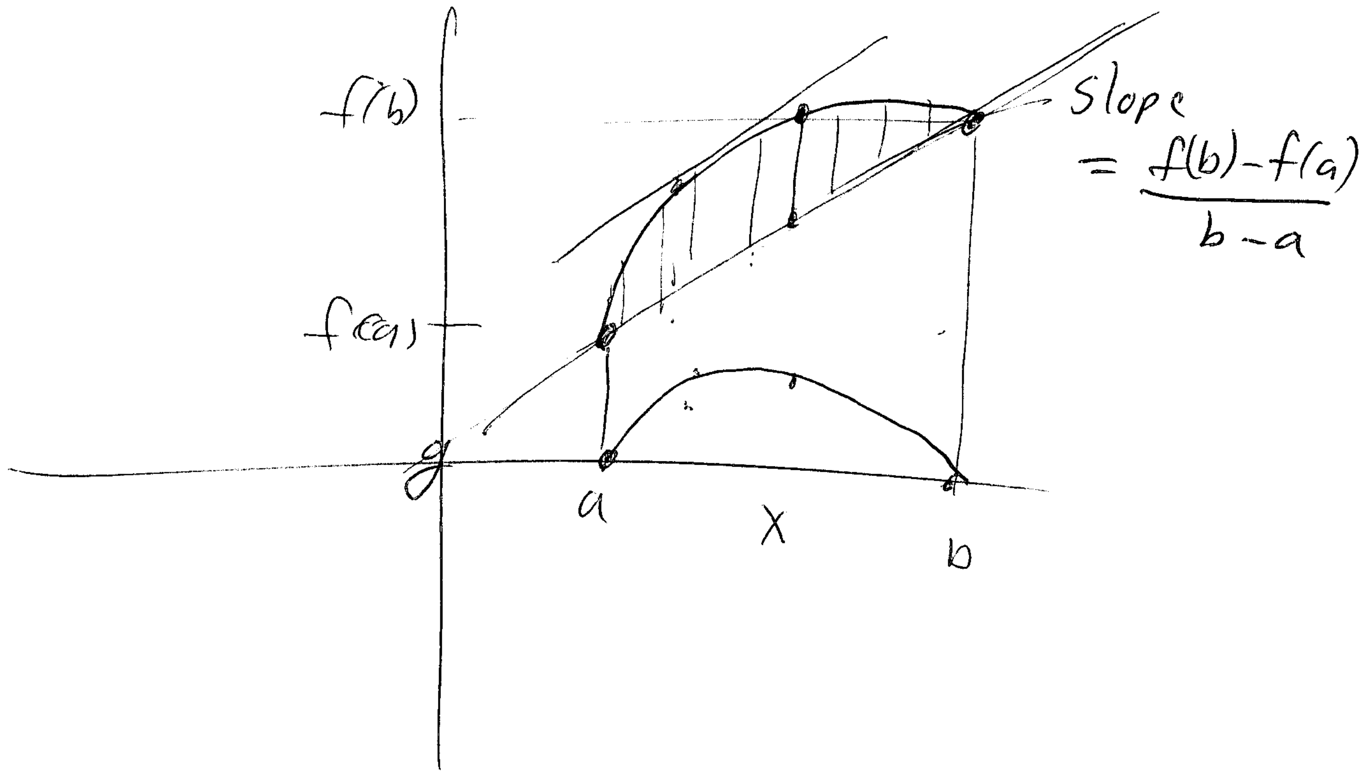
If $f(x) = x^2 + ax + b$

$f'(x) = 2x + a$

Mean Value Thm (MVT)

If f is differentiable on (a,b) and continuous on $[a,b]$, then there is a point c between a and b with

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



$$g(x) = f(a) + (x-a) \cdot \frac{f(b)-f(a)}{b-a}$$

$$h(x) = f(x) - g(x)$$

$$h(a) = 0$$

$$h(b) = 0$$

Rolle's Thm: $h'(c) = 0 \implies f'(c) = g'(c)$

$$= \frac{f(b)-f(a)}{b-a}$$