$$\frac{d}{dx}\left(\sqrt{x^2+1}\right) = ?$$

$f(x) = \sqrt{x} = x^{1/2}$ | $f'(x) = \frac{1}{2\sqrt{x}}$

$g(x) = x^2+1$ | $g'(x) = 2x$

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Chain rule version 1

$$= \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$$

$f(x) = \sin(x)$

$g(x) = \cancel{co}\, x^2$

$$\frac{d}{dx} \sin(x^2) = \cos(x^2) \cdot 2x$$

$$\frac{d}{dx} e^{\sin(x)} = e^{\sin(x)} \cdot \cos(x)$$

$f(x) = e^x$

$g(x) = \sin(x)$

$$\frac{d}{dx} 2^x = \frac{d}{dx}\left(e^{\ln(2)}\right)^x = \frac{d}{dx} e^{x\ln(2)}$$

$$= e^{x\ln(2)} \cdot \ln(2) = 2^x \ln(2)$$

$$\frac{d}{dx} a^x = a^x \ln(a)$$

$y = f(u) = f(g(x))$
$u = g(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ Chain Rule version 2.

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

Suppose $\frac{du}{dx} = 3$ and $\frac{dy}{du} = 2$.

$$\left.\begin{aligned} \Delta u &\approx \frac{du}{dx} \Delta x \\ \Delta y &\approx \frac{dy}{du} \Delta u \end{aligned}\right\} \quad \Delta y \approx \frac{dy}{du} \frac{du}{dx} \Delta x$$

$\cancel{\frac{d}{dx}}$ $\frac{d}{dx}(u^n) = nu^{n-1} \cdot \frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

$\frac{d}{dx}\sin(u) = \cos(u)\frac{du}{dx}$

$\frac{d}{dx}\cos(u) = -\sin(u)\frac{du}{dx}$

$\frac{d}{dx} a^u = a^u \ln(a) \cdot \frac{dy}{dx}$

$u = x^2+1$

$\frac{d}{dx}\sqrt{x^2+1} = \frac{d}{dx} u^{1/2} = \frac{1}{2} u^{-1/2} \cdot \frac{du}{dx}$

$= \frac{1}{2\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$

$u = -t$

$\frac{d}{dt} 2^{-t} = \underbrace{2^{-t}\ln 2}\cdot \underbrace{\frac{d}{dt}(-t)}$

$= -\ln(2)\, 2^{-t} \approx -.69\, 2^{-t}$

$$\frac{d}{dx}\sin\left(\sqrt{x^2+1}\right) = \cos\left(\sqrt{x^2+1}\right)\cdot\frac{d}{dx}\sqrt{x^2+1}$$

$$= \cos\left(\sqrt{x^2+1}\right)\cdot\frac{x}{\sqrt{x^2+1}}.$$

$$\frac{d}{dx}\sin^2\left(\sqrt{x^2+1}\right) = \frac{d}{dx}\left(\sin\left(\sqrt{x^2+1}\right)\right)^2$$

$f(x) = x^2$

$g(x) = \sin\left(\sqrt{x^2+1}\right)$

$$= 2\sin\left(\sqrt{x^2+1}\right)\cdot\frac{d}{dx}\left(\sin\sqrt{x^2+1}\right)$$

$$= 2\sin\sqrt{x^2+1}\cdot\cos\sqrt{x^2+1}\cdot\frac{x}{\sqrt{x^2+1}}$$

$$\lim_{h\to 0} \frac{1-\cos(h)}{h} \cdot \frac{1+\cos(h)}{1+\cos(h)}$$

$$= \lim_{h\to 0} \frac{1-\cos^2(h)}{h(1+\cos(h))} = \lim_{h\to 0} \frac{\sin(h)}{h} \frac{\sin(h)}{1+\cos(h)}$$

$$= 0$$

$$\frac{\cos(x+h)-\cos(x)}{h} = \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cancel{\cos(x)}}{h}$$

$$= \cos(x)\left(\frac{\cos(h)-1}{h}\right) - \sin(x)\frac{\sin(h)}{h}$$