$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(x)$	$f'(x)$
x^n	$n x^{n-1}$
e^x	e^x
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\ln(x)$	$\frac{1}{x}$
$\sin^{-1}(x)$	$1/\sqrt{1-x^2}$
$\tan^{-1}(x)$	$1/(1+x^2)$
$\sec^{-1}(x)$	$1/(x\sqrt{x^2-1})$

product $\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + \left(\frac{du}{dx}\right)v.$

$$\frac{d}{dx}(x^2 e^x) = x^2 \frac{de^x}{dx} + \frac{d}{dx}(x^2) \cdot e^x$$
$$= x^2 e^x + 2x e^x = (x^2+2x)e^x$$

---

quotient. $\frac{d}{dx}\left(\frac{u}{v}\right) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{d}{dx}\left(\frac{3x+5}{\sin(x)}\right) = \frac{\sin(x) \cdot 3 - (3x+5) \cdot \cos(x)}{\sin^2(x)}$$

Chain: $\frac{d}{dx} f(u) = f'(u)\frac{du}{dx}$

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \sin(u) = \cos(u) \frac{du}{dx}$$

$$\frac{d}{dx} \cos(u) = -\sin(u) \frac{du}{dx}$$

$$\frac{d}{dx}\left(\cos(x^2)\right)$$
$$= -\sin(x^2) \cdot 2x$$

$$\frac{d}{dx}\left(x^2\cos\left(\frac{3x+5}{\sin(x)}\right)\right) = x^2\frac{d}{dx}\cos\left(\frac{3x+5}{\sin(x)}\right)$$

$$+ 2x\cos\left(\frac{3x+5}{\sin(x)}\right)$$

$$= -x^2\sin\left(\frac{3x+5}{\sin(x)}\right)\cdot\frac{d}{dx}\left(\frac{3x+5}{\sin(x)}\right)$$

$$+ 2x\cos\left(\frac{3x+5}{\sin(x)}\right)$$

$y = x^n = f(x)$, $f'(a) = \lim_{x \to a} \frac{x^n - a^n}{x - a}$

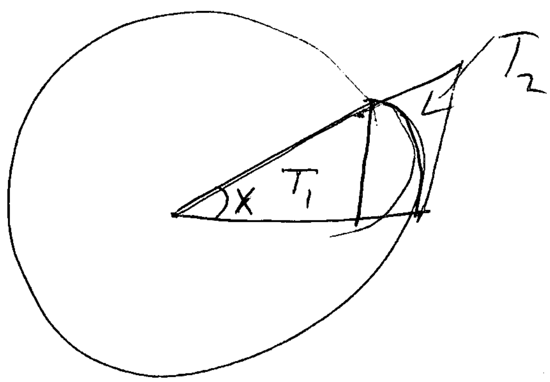
$= \lim_{x \to a} x^{n-1} + a x^{n-2} + a^2 x^{n-3} + \cdots + a^{n-1}$

$= a^{n-1} + a^{n-1} + \cdots + a^{n-1}$

$= n a^{n-1}$

$f'(x) = n x^{n-1}$

$\frac{d}{dx} e^x = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = e^x \lim_{h \to 0} \frac{e^h - 1}{h} = e^x$



$Area(T_1) \leq Area(Wedge) \leq Area(T_2)$

$\frac{1}{2} \sin(x)\cos(x) \leq \frac{x}{2} \leq \frac{1}{2} \frac{\sin(x)}{\cos(x)}$

$\cos(x) \leq \frac{\sin(x)}{x} \leq \frac{1}{\cos(x)}$

$\lim_{x \to 0} \cos(x) \leq \lim_{x \to 0} \frac{\sin(x)}{x} \leq \lim_{x \to 0} \frac{1}{\cos(x)}$

$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

$$\lim_{x \to 0} \frac{\cos(x)-1}{x} = \lim_{x \to 0} \frac{(\cos(x)-1)(\cos(x)+1)}{x \quad (\cos(x)+1)}$$

$$= \lim_{x \to 0} \frac{\cos^2(x)-1}{x(\cos(x)+1)} = \lim -\frac{\sin^2(x)}{x(\cos(x)+1)}$$

$$= \lim_{x \to 0} \left[\underbrace{\left(\frac{\sin(x)}{x}\right)}_{1} \underbrace{\left(\frac{-\sin(x)}{\cos(x)+1}\right)}_{0} \right] = 0$$

$$\frac{d}{dx} \sin(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \left(\sin(x) \underline{\left(\frac{\cos(h)-1}{h}\right)} + \cos(x) \frac{\sin(h)}{h} \right)$$

$$= \sin(x) \cdot 0 + \cos(x) \cdot 1 = \cos(x)$$

$$\frac{d}{dx}\cos(x) = \lim \frac{\cos(x+h)-\cos(x)}{h}$$

$$= \lim \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}$$

$$= \lim \left(\cos(x) \underbrace{\left(\frac{\cos(h)-1}{h} \right)}_{\Rightarrow 0} - \sin(x) \underbrace{\left(\frac{\sin h}{h} \right)}_{\Rightarrow 1} \right)$$

$$= -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \frac{d}{dx}\frac{\sin(x)}{\cos(x)} = \frac{\cos(x)\cdot\frac{d}{dx}\sin(x) - \sin(x)\frac{d}{dx}\cos(x)}{\cos^2(x)}$$

$$= \frac{\cos^2(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = \frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

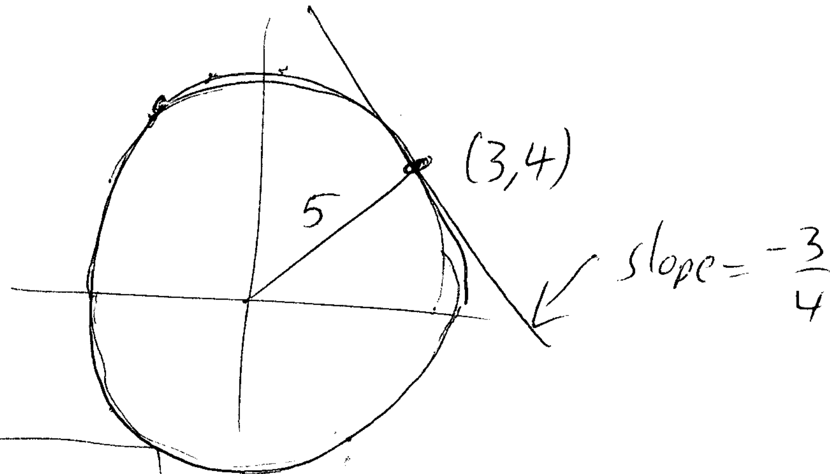
Implicit differentiation

$$\boxed{x^2+y^2=25}$$

$$y^2 = 25 - x^2$$

$$y = \sqrt{25-x^2}$$

$$= (25-x^2)^{1/2}$$



$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25) = 0$$

$$\frac{dy}{dx} = \frac{1}{2}(25-x^2)^{-1/2}\cdot(-2x)$$

$$= \frac{-x}{\sqrt{25-x^2}}$$

$$2x + \frac{d}{dx}y^2 = 0$$

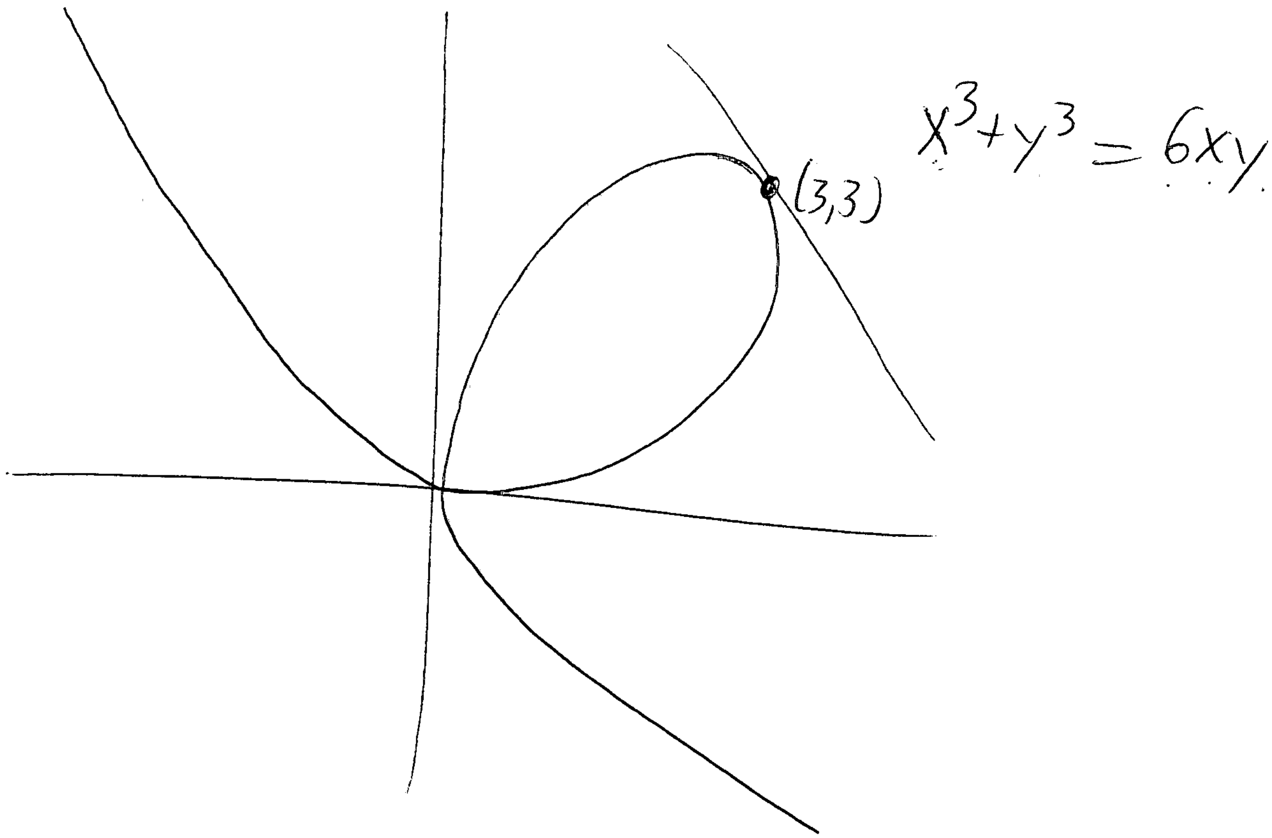
$$2x + 2y\cdot\frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$F(x,y) = 0$$

$$\frac{d}{dx}F(x,y) = 0$$

solve for $\frac{dy}{dx}$



$$3x^2+3y^2\frac{dy}{dx}=6\left(x\frac{dy}{dx}+y\cdot 1\right)$$

$$3x^2+3y^2\,y'=6xy'+6y$$

$$(3y^2-6x)y'=6y-3x^2$$

$$\frac{dy}{dx}=y'=\frac{6y-3x^2}{3y^2-6x}=\frac{2y-x^2}{y^2-2x}$$

$$y = \ln x \iff x = e^y$$

$$1 = e^y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} x^n = \frac{d}{dx}\left(e^{n\ln(x)}\right) = e^{n\ln(x)} \cdot \frac{d}{dx}(n\ln(x))$$

$$= e^{n\ln(x)} \cdot n \frac{1}{x}$$

$$= \frac{x^n \cdot n}{x} = nx^{n-1}$$

$$y = \cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

$$\frac{dy}{dx} = -\frac{d}{dx}\sin^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cot^{-1}(x) = \frac{d}{dx}\left(\frac{\pi}{2} - \tan^{-1}(x)\right) = -\frac{d}{dx}\tan^{-1}(x) = \frac{-1}{1+x^2}$$

$y = \sin^{-1}(x)$

$x = \sin(y)$

$1 = \cos(y) \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} = \frac{1}{\sqrt{1-\sin^2(y)}} = \frac{1}{\sqrt{1-x^2}}$$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos(y) = \sqrt{1-\sin^2(y)}$

$y = \tan^{-1}(x)$

$x = \tan(y)$

$1 = \sec^2(y) \frac{dy}{dx}$

$= \left(1 + \tan^2(y)\right) \frac{dy}{dx}$

$= (1+x^2) \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\tan^{-1}(x^2) = \frac{1}{1+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$$

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\boxed{\frac{d}{dx}\sec^{-1}(x)}$$

$y = \sec^{-1}(x)$

$= \cos^{-1}\left(\frac{1}{x}\right)$

$x = \sec(y) = \frac{1}{\cos(y)}$

$\cos(y) = \frac{1}{x}$

$$\frac{d\sec^{-1}(x)}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2} = \frac{1}{|x|\sqrt{x^2-1}}$$
