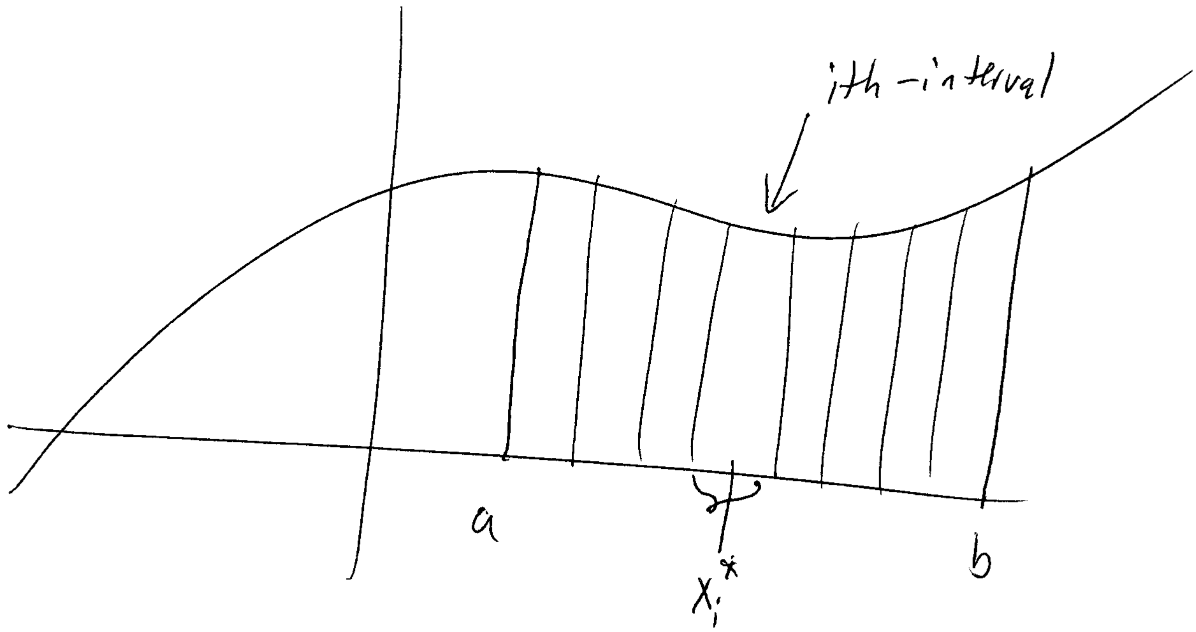
$$\int_a^b f(x)\,dx \overset{\text{def}}{=} \lim_{N \to \infty} \sum_{i=1}^{N} f(x_i^*)\,\Delta x$$



$$\int_1^3 x^2\,dx = \int_1^3 y^2\,dy = \int_1^3 t^2\,dt = \int_1^3 z^2\,dz$$

If velocity $= f(t)$.

Distance traveled between time a and b.

$=$ Distance traveled between $t=a$ and $t=a+\Delta t$

$+$ " " " $t=a+\Delta t$ " $t=a+2\Delta t$

$+$

$\vdots$

$+$ " " " $t=a+(N-1)\Delta t$ and $t=a+N\Delta t$ $=b$

$\approx$ Speed in first interval $\cdot \Delta t$

$+$ " " 2nd " $\cdot \Delta t$.

$+ \cdots$

$+$ Speed in last interval $\cdot \Delta t$

$$\approx f(t_1^*)\Delta t + f(x_2^*)\Delta t + \cdots + f(x_N^*)\Delta t$$

$$= \sum_{i=1}^{N} f(t_i^*)\Delta t$$

Exact distance traveled $= \lim_{N\to\infty} \sum f(x_i^*)\Delta x$

$$\overset{\text{def}}{=} \int_a^b f(t)\,dt$$

Suppose ~~θ~~ velocity $= 20t - 10$

Distance traveled between $t=1$ and $t=3$?

Ans: $\displaystyle\int_1^3 (20t-10)\,dt$

$$\frac{dX(t)}{dt} = \text{velocity} = 20t-10$$

$$X(t) = 10t^2 - 10t + C$$

$$\begin{aligned}
\text{Distance traveled} &= X(3) - X(\cancel{0}1) \\
&= \left(10(3^2) - 10(3) + C\right) \\
&\quad - \left(10(1^2) - 10(1) + C\right) \\
&= 90 - 30 - (10-10) = 60
\end{aligned}$$

Fundamental theorem of calculus. (version 2)

If $F'(x) = f(x)$, then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

$$= F(x)\Big|_a^b$$

$$\int_0^3 e^x\,dx = e^x\Big|_0^3 = e^3 - e^0 = e^3 - 1$$

$$\int_0^{\pi/4} \sin(\theta)\,d\theta = -\cos(\theta)\Big|_0^{\pi/4} = -\cos\left(\frac{\pi}{4}\right) + \cos(0)$$

$$= 1 - \frac{\sqrt{2}}{2}$$

$$\int_1^3 \frac{1}{z}\,dz = \ln(z)\Big|_1^3 = \ln(3) - \ln(1) = \ln(3)$$

$$\int_{-1}^{2} (20t-10)\,dt = 10t^2 - 10t \Big|_{-1}^{2}$$

$$= (10\cdot 4 - 10\cdot 2) - (10+10)$$

$$= 20 - 20 = 0$$

$$\int_{0}^{\pi/2} \sin(\theta)\cos(\theta)\,d\theta = \frac{\sin^2\theta}{2}\Big|_{0}^{\pi/2} = \frac{1}{2} - 0 = \frac{1}{2}$$

$$= -\frac{\cos^2\theta}{2}\Big|_{0}^{\pi/2} = 0 - \left(-\frac{1}{2}\right) = \frac{1}{2}$$

$$\frac{d}{d\theta}\frac{(\sin^2\theta)}{2} = \frac{d}{d\theta}\frac{\left((\sin\theta)^2\right)}{2} = \cancel{2}\sin\theta\cos\theta$$

$$\frac{d}{d\theta}\frac{(\cos^2\theta)}{-2} = \frac{-2\sin\theta\cos\theta}{-2} = \sin\theta\cos\theta$$

$$\int_{-3}^{-1}\frac{1}{x}\,dx = \ln|x|\Big|_{-3}^{-1} = \ln(1) - \ln(3) = -\ln(3)$$

$F(b)-F(a)$

$$\Delta x = \frac{b-a}{N}$$

$$b = a + N\Delta x$$

$$
\begin{aligned}
&= F(a+\Delta x) - F(a) && \big\}\ f(x_1^*)\Delta x \\
&+ F(a+2\Delta x) - F(a+\Delta x) && \big\}\ f(x_2^*)\Delta x \\
&+ F(a+3\Delta x) - F(a+2\Delta x) && \ \ \vdots \\
&+ \\
&\ \ \vdots \\
&+ F(a+N\Delta x) - F(a+(N-1)\Delta x) && \big\}\ f(x_N^*)\Delta x
\end{aligned}
$$

There is a point x_1^* between a and $a+\Delta x$ where $F'(x_1^*) = \dfrac{F(a+\Delta x) - F(a)}{\Delta x}$

$$F(a+\Delta x) - F(a) = F'(x_1^*)\Delta x = f(x_1^*)\Delta x$$

$$F(b)-F(a) = \sum_{i=1}^{N} f(x_i^*)\,\Delta x$$

$$F(b)-F(a) = \lim_{N\to\infty} \sum_{i=1}^{N} f(x_i^*)\,\Delta x \overset{def}{=} \int_a^b f(x)\,dx$$

Definite integrals $\xleftrightarrow{\text{Fundamental thm, version 2.}}$ Anti-derivatives.

$\int_a^b f(x)\,dx$ $\qquad$ $F(x)$ with $F'(x) = f(x)$

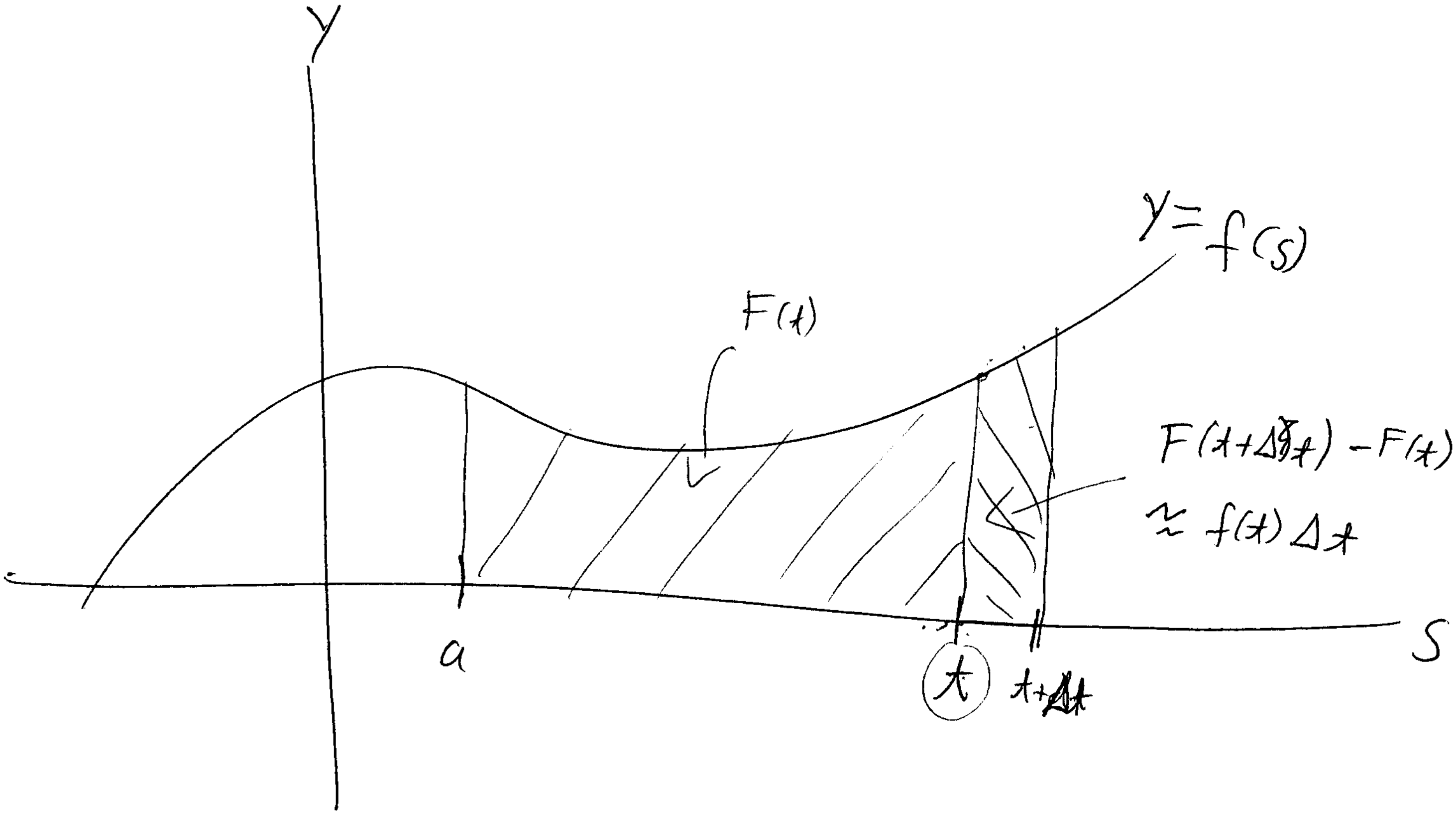
FTC, version 1.

$\int_a^x f(s)\,ds$ = Indefinite integral

Velocity $= f(t)$ $\qquad$ Start at time $t=a$.

Distance traveled by time t. is

$F(t) = \int_a^t f(s)\,ds$ $\qquad$ Indefinite integral of f,



$$F(t) = \int_a^t f(s)\,ds = \text{~~tot~~ total distance traveled by time } t$$

$$= \text{Area ~~of~~ under the curve up to } t.$$

~~Fund~~ FTC – version 1

$$\boxed{\frac{d}{dt}\int_a^t f(s)\,ds = f(t)}$$

$$\frac{F(t+\Delta t) - F(t)}{\Delta t} \approx f(t)$$

$$F'(t) = f(t)$$

$$\frac{d}{dx}\int_{1}^{x}\underbrace{\left(e^{s}+3s^{3}+\frac{\tan^{-1}(s)}{s^{2}}\right)}_{f(s)}ds$$

$$=\frac{d}{dx}\int_{1}^{x}f(s)\,ds = f(x) = e^{x}+3x^{3}+\frac{\tan^{+1}(x)}{x^{2}}$$

$$\frac{d}{dt}\int_{t}^{7}e^{s}\,ds = e^{s}\Big|_{t}^{7} = e^{7}-e^{t}$$

$$=\frac{d}{dt}\left(-\int_{7}^{t}e^{s}\,ds\right) = -e^{t}$$

$$\frac{d}{dx}\int_1^{x^2} e^t\,dt \qquad u = x^2$$

$$\frac{d}{dx}\int_1^{u} e^t\,dt = \frac{d\int_1^u e^t\,dt}{du}\cdot\frac{du}{dx}$$

$$= e^u \cdot \frac{du}{d\cancel{x}} = e^{x^2}\cdot 2x$$

$$\frac{d}{dx}\int_{x^2}^{e^x}(2t\,dt) = \frac{d}{dx}\left(\int_a^{e^x} - \int_a^{x^2}\right)2t\,dt$$

$$= 2e^x\frac{d}{dx}e^x - 2(x^2)\cdot\frac{dx^2}{dx}$$

$$= 2e^{2x} - 4x^3$$

$$= \frac{d}{d\cancel{x}}\,t^2\Big|_{x^2}^{e^x} = \frac{d}{dx}\left((e^x)^2-(x^2)^2\right) = \frac{d}{dx}\left(e^{2x}-x^4\right)$$

$$= 2e^{2x} - 4x^3$$

$$\boxed{\frac{d}{dx}\int_{g(x)}^{h(x)} f(s)\,ds = f(h(x))\cdot h'(x) - f(g(x))\cdot g'(x)}$$

FTC - version 1 talks about <u>indefinite integrals</u>

$$\frac{d}{dx}\int_a^x f(s)\,ds = f(x)$$

"The derivative of the indefinite integral is the original function"

(Combine w/ chain rule to get

$$\frac{d}{dx}\int_{g(x)}^{h(x)} f(s)\,ds$$
)

FTC - version 2 relates anti-derivatives ~~an~~ and <u>definite</u> integrals.

$$\int_a^b F'(x)\,dx = F(b) - F(a).$$

Integral of the ~~derivative~~ derivative is the original function.

Horrible Notation Alert:

$$\int f(x)\,dx \quad \text{means} \quad \int_{a}^{x} f(s)\,ds$$

defined up to constant

= anti-derivative of $f(x)$.