f vs t

1800
1600
1400
1200
1000
800
600
400
200
0
Jan
Feb
March
April
May
June
July
Aug
Sep
Oct
Nov
Dec
Jan
Feb
March
April
May
June
July
Aug
Sep
Oct
Nov
Dec
Jan
Feb
March
April
May
June
July
Aug
Sep
Oct
a
b
2009
2010
2011

F vs t

$f(t) =$ rate of electricity use at time t at the Sadun house.

Power usage between time a and time b.

$$= \int_a^b f(t)\,dt = \lim_{N\to\infty}\left(\sum_{i=1}^{N} f(x_i^*)\Delta x\right)$$

$F(t) =$ reading on power meter at time t.

$$\frac{dF(t)}{dt} = f(t).$$

F is an anti-derivative of f.

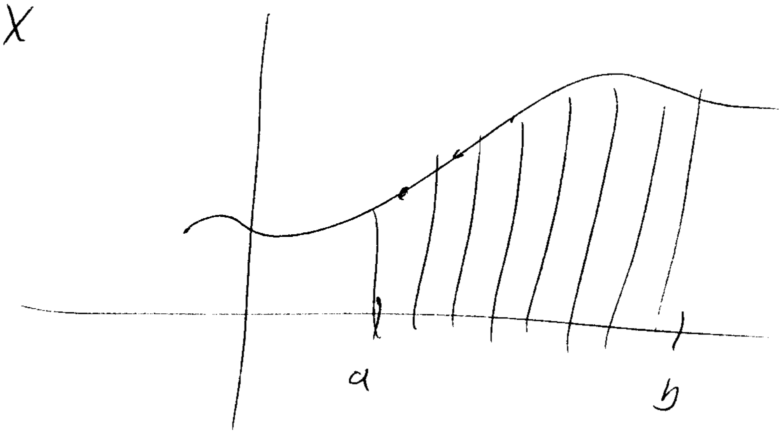
$$I(t) = \int_{\text{Jan } 2009}^{t} f(s)\,ds$$

= power used between Jan 2009 and t.

Given $f(x)$

1) Definite integral $\int_a^b f(x)\,dx = \int_a^b f(s)\,ds = \int_a^b f(t)\,dt$

$= \lim_{N \to \infty} \sum_{i=1}^{N} f(x_i^*)\Delta x$



$\Big\updownarrow$ FTC2

2) $F(x) =$ function with $F'(x) = f(x)$

$\Big\updownarrow$ FTC 1

3) $\int_a^x f(s)\,ds$ = Indefinite integral
= running total.

FTC 1: The indefinite integral is an anti-derivative.

$$\frac{d}{dx} \int_a^x f(s)\,ds = f(x)$$

FTC 2: If $F' = f$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$I(t) = \int_a^t f(s)\,ds.$$

$f(t) =$ power usage

$F(t) =$ power meter.

FTC 1 $\Rightarrow$ FTC 2

$$\frac{d}{dx}\int_a^x f(s)\,ds = f(x)$$

$$\frac{d}{dx}F(x) = f(x)$$

$$\frac{d}{dx}\left(\int_a^x f(s)\,ds - F(x)\right) = f(x) - f(x) = 0$$

$$\int_a^x f(s)\,ds = F(x) + C$$

$$0 = \int_a^a f(s)\,ds = F(a) + C \Rightarrow C = -F(a)$$

$$\int_a^x f(s)\,ds = F(x) - F(a)$$

$$\int_a^b f(s)\,ds = F(b) - F(a)$$

FTC2 $\rightarrow$ FTC1

$$\int_a^b f(s)\,ds = F(b) - F(a) \qquad \text{for all } a, b$$

$$\int_a^x f(x)\,ds = F(x) - F(a) \qquad \text{for all } x$$

$$\frac{d}{dx}\int_a^x f(s)\,ds = F'(x) = f(x) \qquad \blacksquare$$

Many authors call the anti-derivative an indefinite integral.

$$\int x^2\,dx = \frac{x^3}{3} + C$$

$$\frac{d}{dx}\int_{x^2}^{x^3} f(s)\,ds = \frac{d}{dx}\left(F(x^3) - F(x^2)\right)$$

$$= F'(x^3)\cdot 3x^2 - F'(x^2)\,2x$$

$$= 3x^2 f(x^3) - 2x\,f(x^2)$$

Limits (Close is good enough)

$$\lim_{x \to a} f(x) = L \quad \text{means}$$

"When x is close to a (but $\neq a$),
$f(x)$ is close to (or $=$) L."

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} (x+1) = 2.$$

$x \to a^+$ when x is slightly $> a$
$x \to a^-$ " " " " $< a$

$x \to \infty$ when x is large and positive.
$x \to -\infty$ " " " " " negative.

$$\lim_{x \to a} f(x) = \infty \quad \text{means}$$

"When $x \approx a$ (but $\neq a$), $f(x)$ is large & positive"

$\lim (f \pm g) = \lim f \pm \lim g$ (except $\infty - \infty$)

$\lim (fg) = (\lim f)(\lim g)$ (except $0 \cdot \infty$)

$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$ when $\lim g \neq 0$ (or $\frac{\infty}{\infty}$)

If ~~lim~~ $\frac{f}{g} \Rightarrow \frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim \left(\frac{f}{g}\right) = \lim \left(\frac{f'}{g'}\right)$$

L'Hôpital's rule

f is continuous at a if

$$\lim_{x\to a} f(x) = f(a)$$

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1) If f and g are cont, so are
$f \pm g$, fg, $f(g(x))$, $\frac{f}{g}$ (where $g \neq 0$)

2) Polynomials are cont.

3) e^x and $\ln(x)$ are cont (where defined)

4) $\sin(x)$, $\cos(x)$ are cont.

5) $\tan(x)$ cont except where $\cos(x)=0$
$\cot(x)$ " " " $\sin(x)=0$

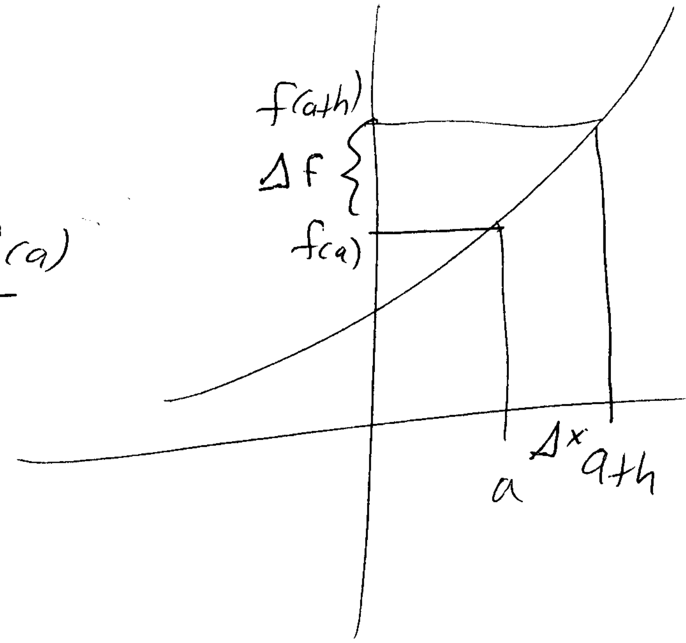
If $f(x)$ is cont, ~~at g(a)~~

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim \frac{\Delta f}{\Delta x}$$



$$= \lim \frac{\text{change in output}}{\text{change in input}}$$

$$\lim_{x \to e} \frac{\ln(x) - 1}{x - e} = f'(e)$$

$f(x) = \ln(x)$
$f(e) = 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$