

Total length of fence = 2400

$$2y + X = 2400$$

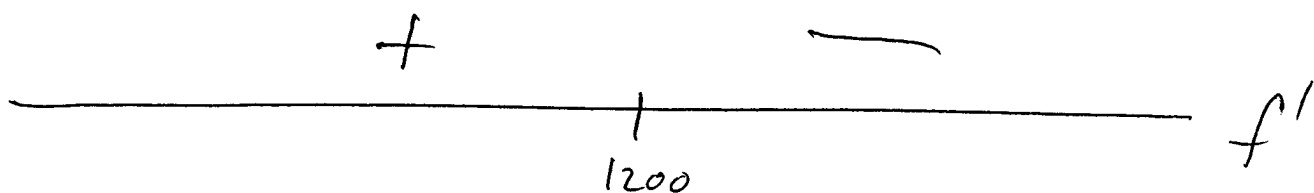
$$X = \frac{2400 - 2y}{1}$$

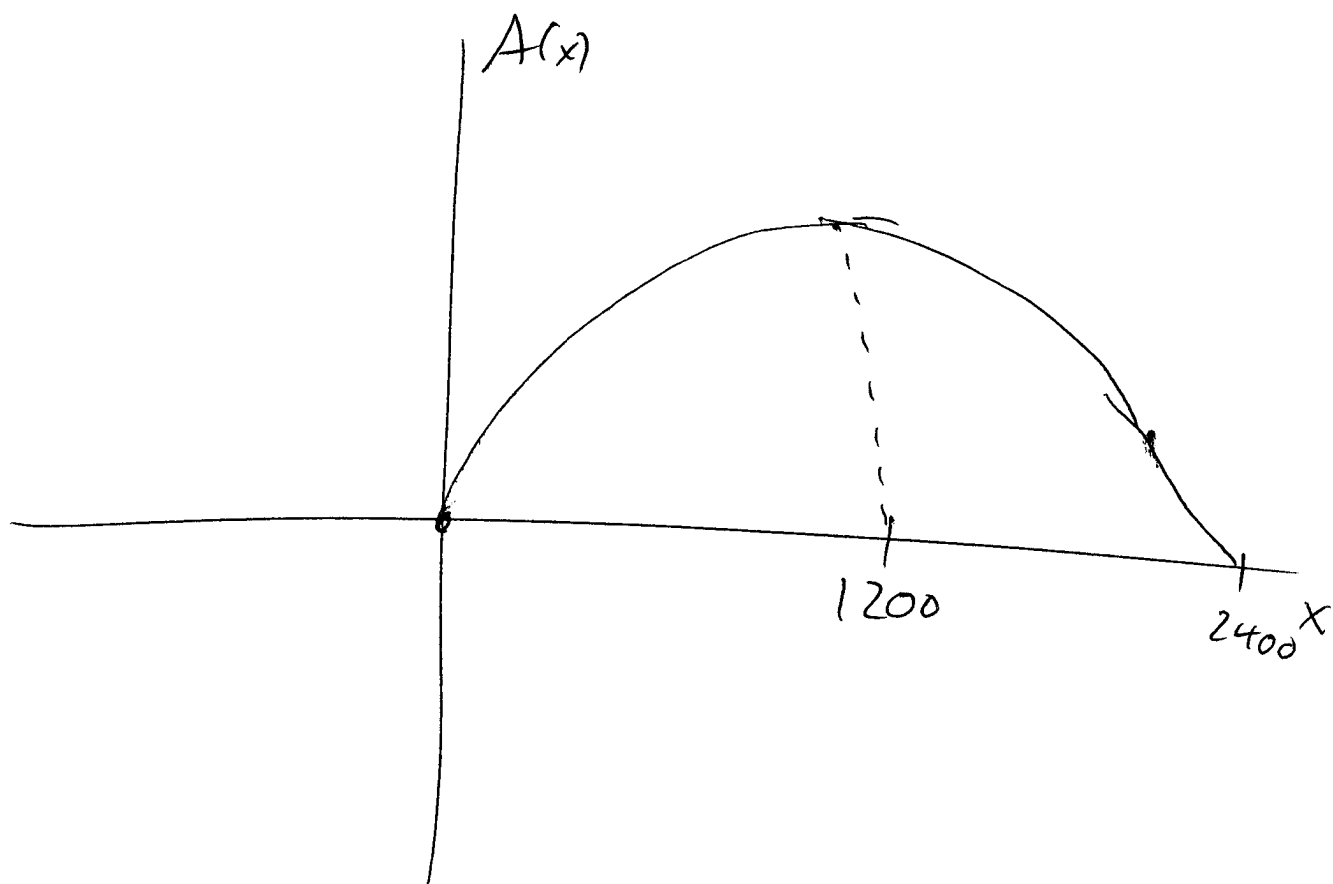
$$y = \frac{2400 - X}{2}$$

$$A(x) = xy = x \left(\frac{2400 - x}{2} \right) = 1200x - \frac{x^2}{2}$$

Find critical pts.

$$0 = A'(x) = 1200 - x \Rightarrow \begin{aligned} x &= 1200 \\ y &= 600 \end{aligned}$$



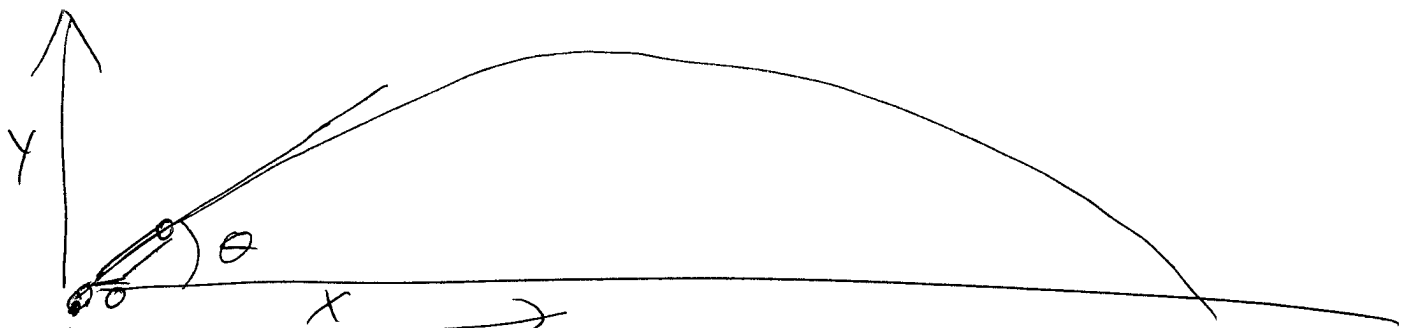


First derivative rule: If

$\begin{array}{c} + \qquad | \qquad - \\ \hline \qquad \qquad a \end{array} f'$, then
 a is an absolute max

If

$\begin{array}{c} - \qquad | \qquad + \\ \hline \qquad \qquad a \end{array} f'$, then
 a is an absolute min



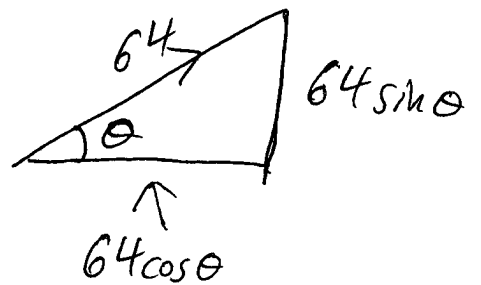
Shoot at speed 64 ft/sec

How to shoot as far as possible?

Initially, $x=y=0 \Rightarrow x(0)=y(0)=0$

$$\frac{dx}{dt}(0) = 64 \cos \theta$$

$$\frac{d^2x}{dt^2} = 0$$



$$x(t) = (64 \cos \theta) t$$

$$\frac{dy}{dt}(0) = 64 \sin \theta$$

$$\frac{d^2y}{dt^2} = -32$$

$$\frac{dy}{dt} = -32t + C$$

$$C = 64 \sin \theta$$

$$\frac{dy}{dt}(0) = -32(0) + C = 64 \sin \theta$$

$$\frac{dy}{dt} = 64 \sin \theta - 32t$$

at $t = 2 \sin \theta$, top of the arc

at $t = 4 \sin \theta$ hit ground.

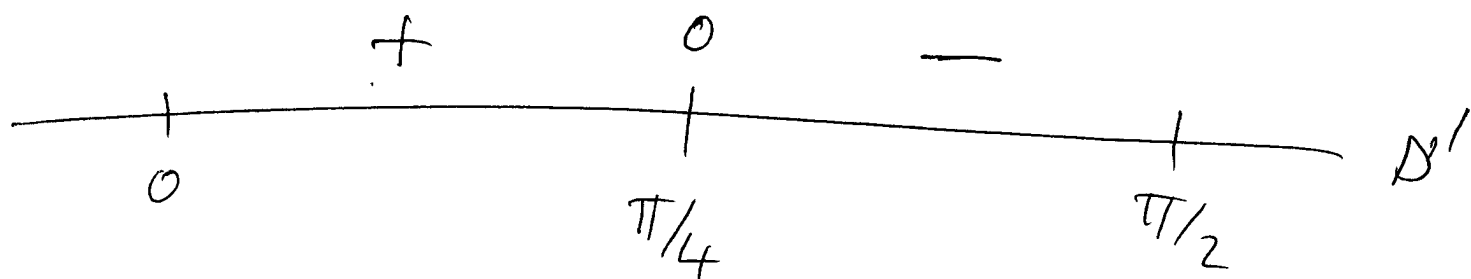
at $t = 4 \sin \theta$, $x = 64 \cos \theta t = 256 \cos \theta \sin \theta$

$$D(\theta) = 256 \sin \theta \cos \theta$$

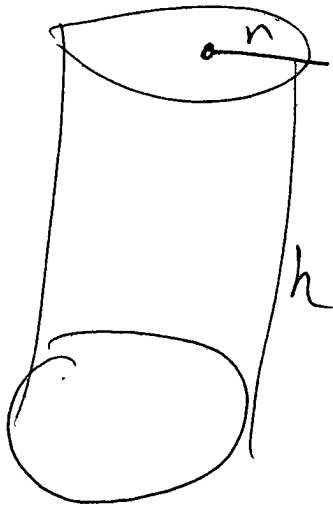
Find crit. pt.

$$D'(\theta) = 256 (\cos \theta \cos \theta + \sin \theta (-\sin \theta))$$

$$= 256 (\cos^2 \theta - \sin^2 \theta) = 256 \cos(2\theta)$$



$$D\left(\frac{\pi}{4}\right) = 256 \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2} = 128$$



Make a cylindrical can that holds 1L, using as little metal as possible.

$$\text{Area of top} = \pi r^2$$

$$\text{Area of bottom} = \pi r^2$$

$$\begin{aligned} \text{Area of side} &= h \times (\text{circumference}) \\ &= 2\pi r h \end{aligned}$$

Minimize $2\pi r^2 + 2\pi r h$

Volume = $\pi r^2 h = 1$ $h = \frac{1}{\pi r^2}$

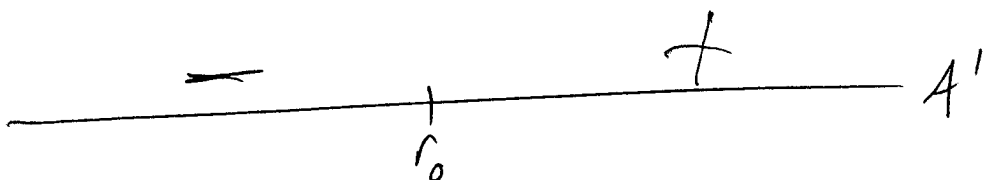
$$\text{Area} = 2\pi r^2 + 2\pi r \cdot \frac{1}{\pi r^2} = 2\pi r^2 + \frac{2}{r} = A(r)$$

$$A'(r) = 4\pi r - \frac{2}{r^2} = 0 \quad \Rightarrow \quad 4\pi r^3 = 2$$

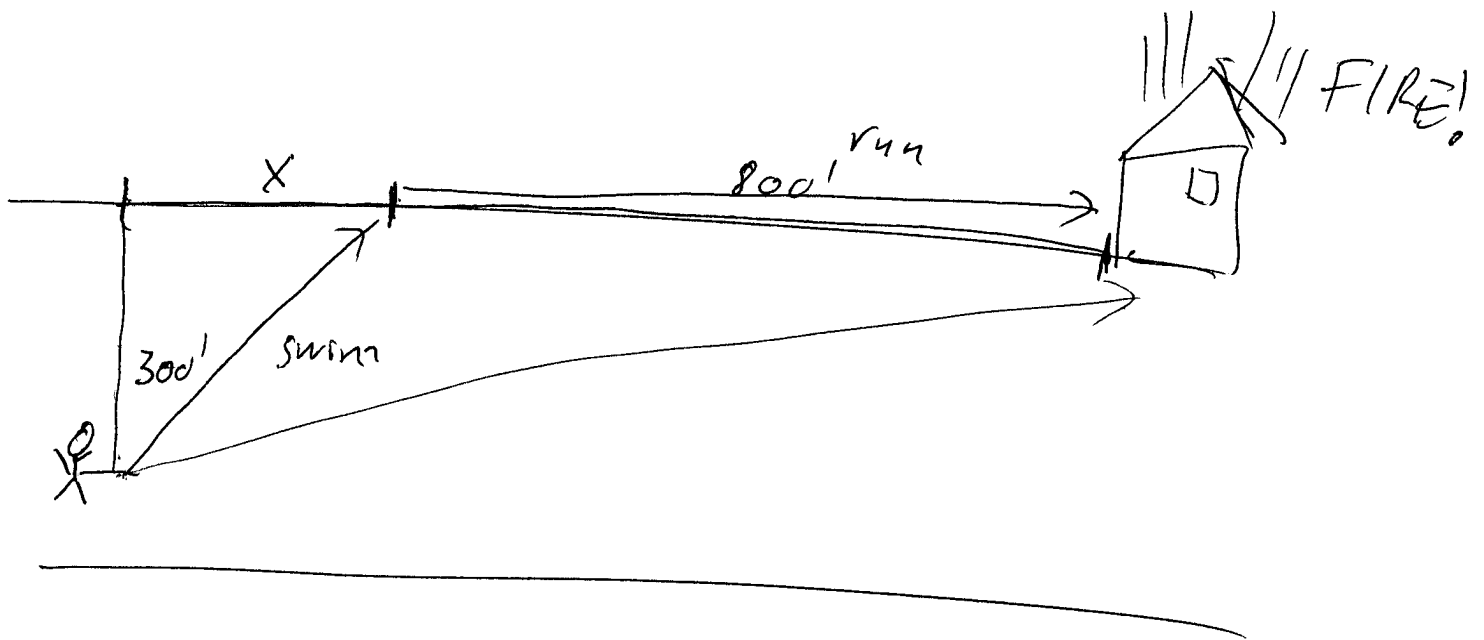
$$r^3 = \frac{1}{2\pi}$$

$$r_0 = \sqrt[3]{\frac{1}{2\pi}} = r_0$$

$$h = \frac{1}{\pi r^2} = \frac{r}{\pi r^3} = \frac{2r}{\pi}$$



- 1) Define your variables
- 2) Express everything in terms of those variables
- 3) Use info to eliminate all but one variable. Express goal in terms of that variable.
- 4) Take derivative and set $= 0$.
- 5) If necessary make sign chart of f' .



swims at 6' / sec
 runs at 8' / sec

$$\text{swim distance} = \sqrt{300^2 + x^2}$$

$$\text{swim time} = \frac{\sqrt{300^2 + x^2}}{6}$$

$$\text{run distance} = 800 - x$$

$$\text{run time} = \frac{800 - x}{8}$$

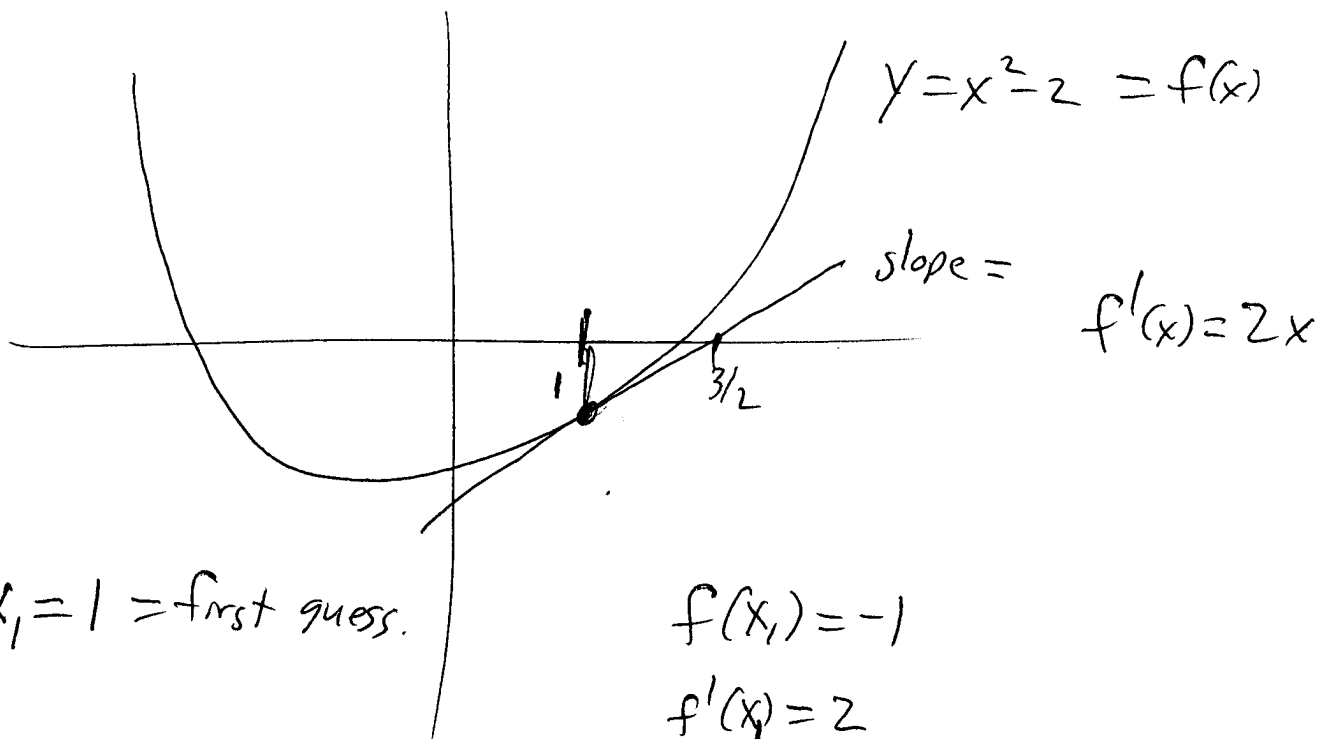
$$\text{total time} = \frac{\sqrt{300^2 + x^2}}{6} + \frac{800 - x}{8} = f(x)$$

$$\text{Rate} \times \text{time} = \text{distance}$$

$$\text{time} = \frac{\text{distance}}{\text{rate}}$$

Find $\sqrt{2}$

Find a solution to $x^2 - 2 = 0$



$$x_1 = 1 = \text{first guess.}$$

$$f(x_1) = -1$$

$$f'(x_1) = 2$$

$$x_2 = 1 + \frac{1}{2} = \frac{3}{2}$$

$$f(x_2) = 1/4$$

$$f'(x_2) = 3$$

$$x_3 = \frac{3}{2} - \frac{(1/4)}{3} = \frac{3}{2} - \frac{1}{12} = \frac{17}{12}$$

$$f\left(\frac{17}{12}\right) = \frac{1}{144}$$

$$x_4 = \frac{17}{12} - \frac{1/144}{17/6} = \frac{577}{408}$$

$$f'\left(\frac{17}{12}\right) = \frac{17}{6}$$

$$\approx 1.414215686$$

$$x_5 = \frac{577}{408} - \frac{(\text{overshoot})}{f'(x_5)} = \frac{665857}{470832}$$

Idea: Find solutions to
 $f(x)=0$

First guess = x_1

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

