1. What is a derivative?

2. How do you compute it?

3. What is it good for?

What is it?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

= rate of change

= extra output / extra input.

= slope of tangent line to graph.

= velocity, if $f(t)$ = position.

~~conve~~

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

How do you compute it?

1. Basic formulas.

$$\frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{du^n}{dx} = nu^{n-1}\frac{du}{dx}$$

$$\frac{d\sin(x)}{dx} = \cos(x)$$

$$\frac{d}{dx}\sin(u) = \cos(u)\frac{du}{dx}$$

$$\frac{d\cos(x)}{dx} = -\sin(x)$$

$$\frac{d}{dx}\cos(u) = -\sin(u)\frac{du}{dx}$$

$$\frac{de^x}{dx} = e^x$$

$$\frac{d}{dx}e^u = e^u\frac{du}{dx}$$

$$\frac{d\ln|x|}{dx} = \frac{1}{x}$$

$$\frac{d}{dx}\ln|u| = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d\tan^{-1}(x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\tan^{-1}(u) = \frac{du/dx}{1+u^2}$$

$$\frac{d\sin^{-1}(x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sin^{-1}(u) = \frac{du/dx}{\sqrt{1-u^2}}$$

(tan, cot, sec, csc)

2. Products & quotients

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = f'(x)\,g(x) + f(x)\,g'(x)$$

$$\frac{d}{dx}\,\frac{f(x)}{g(x)} = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{g(x)^2} \neq \frac{f'(x)}{g'(x)}$$

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3. Chain rule

$$\frac{d}{dx}\,f(u) = f'(u)\,\frac{du}{dx}$$

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4. Mix & match.

$$\frac{d}{dx}\,\tan^{-1}\left(\frac{3x}{x^2+7}\right) = \frac{1}{1+\left(\frac{3x}{x^2+7}\right)^2}\cdot\frac{d}{dx}\left(\frac{3x}{x^2+7}\right)$$

$$\frac{d}{dx} \sin^3(x^2+7)$$

$$= \frac{d}{dx} \left(\sin(x^2+7)\right)^3 = 3 \left(\sin(x^2+7)\right)^2 \cdot \frac{d}{dx} \sin(x^2+7)$$

$$= 3 \sin^2(x^2+7) \cos(x^2+7) \frac{d}{dx} (x^2+7)$$

$$= 6x \sin^2(x^2+7) \cos(x^2+7)$$

5. Implicit differentiation.

Take derivatives of <u>equations</u>

Ex: $\cancel{x}e^y + y^2 + x^2 = 17$

$$e^y y' + 2yy' + 2x = 0$$

$$(e^y + 2y)y' = -2x$$

$$y' = \frac{-2x}{e^y + 2y}$$

6. Logarithmic differentiation.

If $\ln(f(x))$ is simpler than $f(x)$,

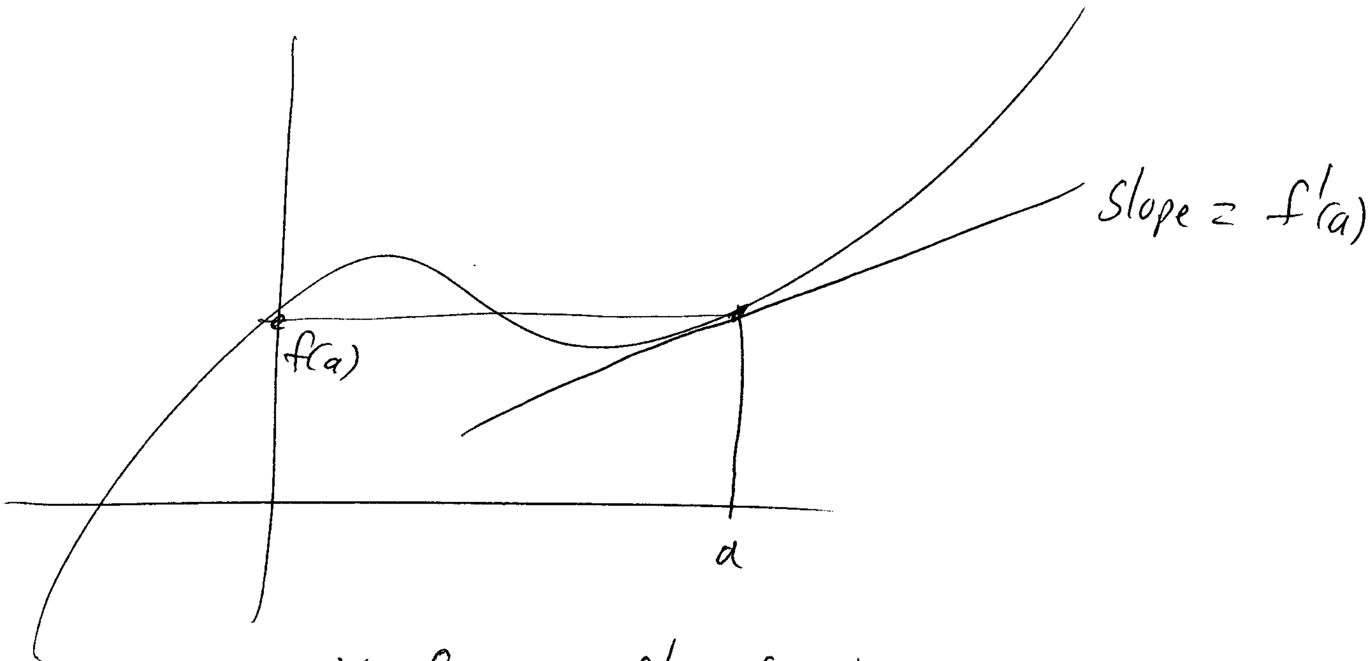
$(y = f(x))$

$$\frac{d}{dx}\ln(y) = \frac{1}{y}\frac{dy}{dx}$$

$$\frac{dy}{dx} = y\,\frac{d}{dx}\ln(y)$$

Things to do with f'

1. Find a tangent line



$$y - f(a) = f'(a)(x-a)$$

~~If Δx is small~~ $\quad f'(a) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$

If Δx is small, $\quad f'(a) \approx \frac{\Delta f}{\Delta x}$

$$\Delta f \approx f'(a)\,\Delta x$$ differentials

$$\Delta x \approx \Delta f / f'(a)$$ Newton's method.

Ex:

$\cancel{f(x)}$ Find $\tan^{-1}(.03)$

$f(x) = \tan^{-1}(x)$ $\qquad a = 0$ $\qquad \tan^{-1}(0) = 0 = f(0)$

$f'(x) = \dfrac{1}{1+x^2}$ $\qquad f'(0) = 1$

Tangent line is $\dfrac{y-0}{x-0} = 1 \implies y = x$

If $x = .03$, $y = .03$ on tangent line,

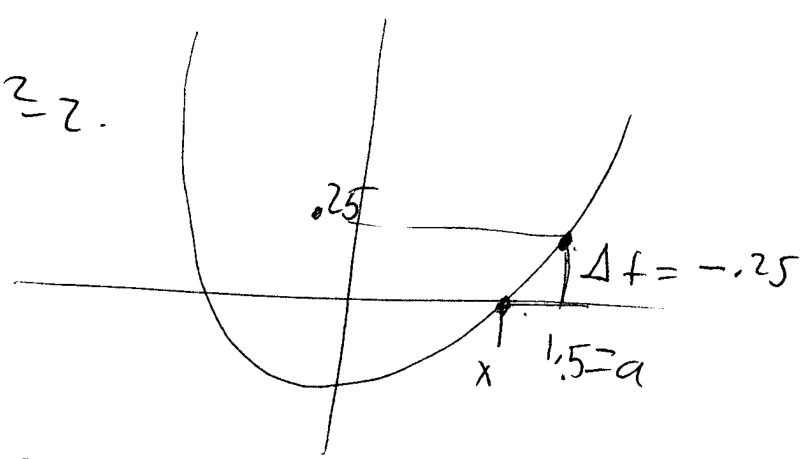
$f(.03) \approx .03$

Ex: Find $\sqrt{2}$ from fact that $(1.5)^2 = 2.25$

$f(x) = x^2 = 2.$

$f'(x) = 2x$

$f'(a) = 3$



$f'(a) =$

$\dfrac{\Delta f}{\Delta x} \approx 3$ $\qquad \Delta x \approx \dfrac{\Delta f}{3} = \dfrac{-1/4}{3} = -1/12$

$x \approx 1.5 - \frac{1}{12}$

$= 1.41\overline{6}666$

2. Related rates

If x and y are related by some formula, then so are $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

Ex If $x^2+y^2=25$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0.$$

$$\frac{dy}{dt}=\frac{-x}{y}\frac{dx}{dt}$$

3. Max-min.

"What goes up has to stop before it comes down".

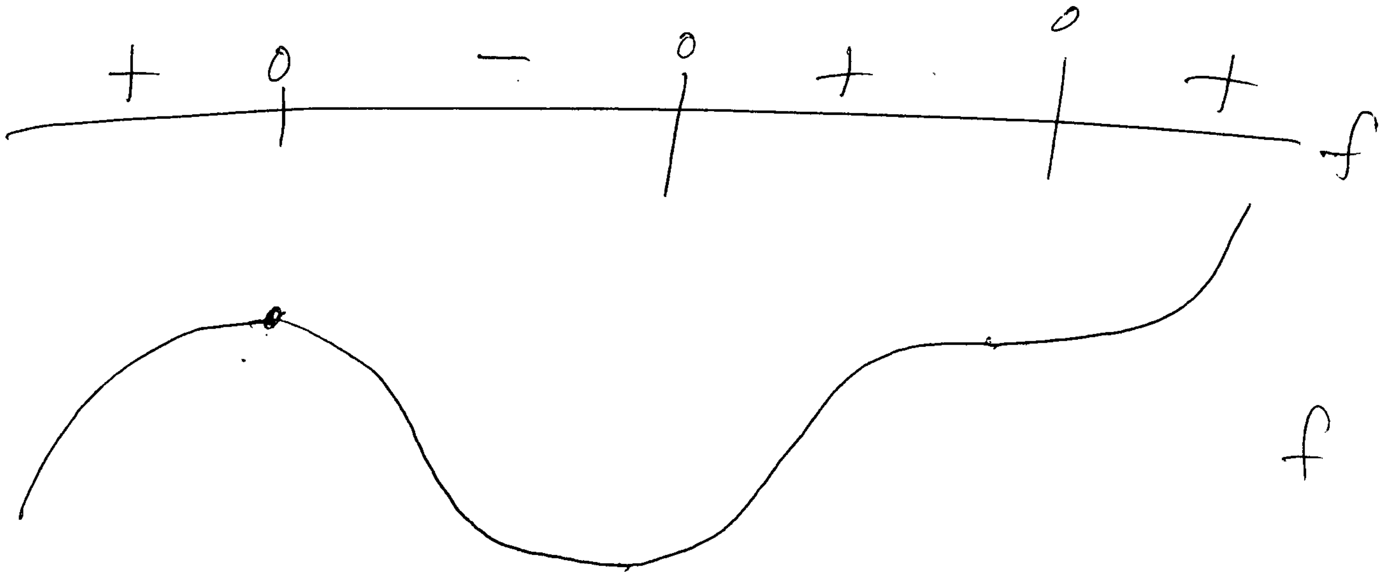
At a local maximum, either

1) $f'(x)=0$
2) $f'(x)$ DNE, or
3) Endpoint.

If $f(x)$ is continuous on $[a,b]$, there is a maximum.

1) List the critical pts. (critical #s)
2) List the endpoints
3) Compare.

Sign charts for f'

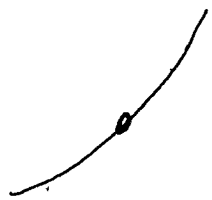
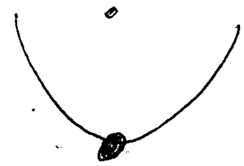
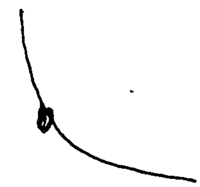
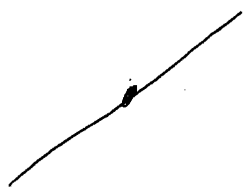
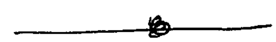
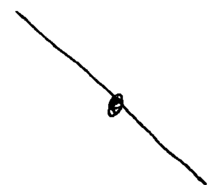
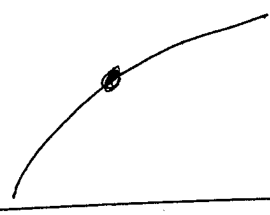
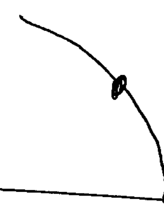


At a local max, f' goes from + to −

" " " min, f' " " − to +

If $f'(a)=0$ and $f''(a)>0$, local min

If $f'(a)=0$ and $f''(a)<0$, local max

Sign of f' / Sign of f''	$+$	0	$-$
$+$			
0			
$-$			

L'Hôpital's Rule

Used to evaluate $\frac{0}{0}$ or $\frac{\infty}{\infty}$ limits.

$$\text{Lim } \frac{f}{g} = \text{Lim } \frac{f'}{g'}$$

$$\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{3x^2}{2x} = \frac{3}{2}$$

$$\lim_{x\to 1} \frac{x^3-1}{x^2+1} = \frac{0}{2} = 0$$

$0 \cdot \infty$ if $f(x) \to 0$, $g(x) \to \infty$

$\frac{1}{f(x)} \to \infty$ $\quad \frac{1}{g}(x) \to 0$

$$f \cdot g = \frac{f}{1/g} = \text{"}\frac{0}{0}\text{"}$$

0^0, $\cancel{\infty}$, 1^∞, ∞^0 problems.

$$\lim_{x\to\infty}\left(\frac{1}{x}\right)^{1/x^2} = y$$

$$\ln y = \lim_{x\to\infty} \frac{1}{x^2}\ln\left(\frac{1}{x}\right) = \lim \frac{\ln(1/x)}{x^2}$$

$$= \lim \frac{-\ln(x)}{x^2}$$

$$= \lim \frac{-1/x}{2x} = 0$$

$\ln y \Rightarrow 0$

$y \Rightarrow e^0 = 1$