Useful Rules

1) $\frac{d}{dx} x^n = n x^{n-1} \qquad (n = 0, 1, 2, 3, \dots)$

2) $\frac{d}{dx} e^x = e^x$

3) $\frac{d}{dx} a^x = a^x \left(\lim_{h \to 0} \frac{a^h - 1}{h} \right)$

4) $\frac{d}{dx} c = 0$

5) $\frac{d}{dx} (f \pm g) = f' \pm g'$

6) $\frac{d}{dx} (cf) = c f'$

$$\left(\frac{d}{dx}(3x^2 + 7x - 9) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(7x) - \frac{d}{dx}(9) \right.$$

$$= 3 \frac{d}{dx} x^2 + 7 \frac{d}{dx} x - \frac{d}{dx} 9$$

$$\left. = 3(2x) + 7 \cdot 1 - 0 = 6x + 7 \right)$$

$f(x) = a^x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim \frac{a^{x+h} - a^x}{h}$$

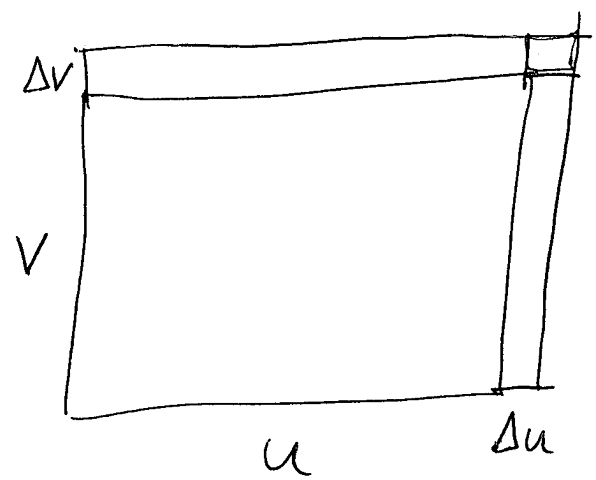
$$= \lim a^x \frac{(a^h - 1)}{h} = a^x \lim_{h \to 0} \left(\frac{(a^h - 1)}{h} \right)$$

Product rule

$$f(x) = u \cdot v \qquad \left(\text{eg. } f(x) = \underset{u}{(x^2+1)}\,\underset{v}{e^x}\right)$$

$$\frac{df}{dx} = \frac{du}{dx} \cdot v + u\,\frac{dv}{dx}$$

$$= 2x\,e^x + (x^2+1)\,e^x = (x^2+2x+1)\,e^x$$



Area $= f$

$$\Delta f = \Delta u \cdot v + \Delta v \cdot u + \Delta u \cdot \Delta v$$

$$\frac{\Delta f}{\Delta x} = v\,\frac{\Delta u}{\Delta x} + u \cdot \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta x} \cdot \Delta v$$

$$\frac{df}{dx} = v \cdot \frac{du}{dx} + u\,\frac{dv}{dx} + \frac{du}{dx}\,0$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{u(x)V(x) - u(a)V(a)}{x - a}$$

$$= \lim_{x \to a} \frac{u(x)V(x) - u(x)V(a) + u(x)V(a) - u(a)V(a)}{x - a}$$

$$= \lim_{x \to a} u(x)\left(\frac{V(x) - V(a)}{x - a}\right) + \lim_{x \to a} \left(\frac{u(x) - u(a)}{x - a}\right)V(a)$$

$$= u(a)\, V'(a) + u'(a)V(a)$$

$$\frac{dx}{dx} = 1$$

$$\frac{d(x^2)}{dx} = x\,\frac{dx}{dx} + \frac{dx}{dx}\cdot x = x + x = 2x$$

$$\frac{d(x^3)}{dx} = \frac{d}{dx}(x^2\cdot x) = x^2\frac{dx}{dx} + \frac{dx^2}{dx}\cdot x = x^2 + 2x\cdot x$$
$$= 3x^2$$

If $f(x) = \dfrac{u}{v}$, $\dfrac{df}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$f = \left(\frac{f}{g}\right)\cdot g \qquad f' = \left(\frac{f}{g}\right)'\cdot g + \left(\frac{f}{g}\right)\cdot g'$$

$$\left(\frac{f}{g}\right)'\cdot g = f' - \left(\frac{f}{g}\right)\cdot g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g}{g^2} - \frac{f\cdot g'}{g^2} = \frac{f'\cdot g - f\cdot g'}{g^2}$$

$$\frac{d}{dx}\left(\frac{x^2+1}{x+7}\right) = \frac{(x+7)(2x)-(x^2+1)\cdot 1}{(x+7)^2}$$

$$= \frac{2x^2+14x-x^2-1}{(x+7)^2} = \frac{x^2+14x-1}{(x+7)^2}$$

$$\frac{d}{dx}(x^{-5}) = \frac{d}{dx}\left(\frac{1}{x^5}\right) = \frac{x^5\frac{d}{dx}1 - 1\cdot\frac{dx^5}{dx}}{x^{10}}$$

$$= \frac{-5x^4}{x^{10}} = -5x^{-6}$$

$$\frac{dx^n}{dx} = nx^{n-1}$$

~~$n = 1, 2, 3, \dots$~~

ANY real n.

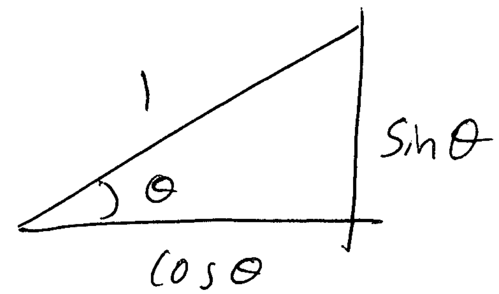
$$x = \sqrt{x}\cdot\sqrt{x} \qquad \frac{dx}{dx} = \sqrt{x}\,\frac{d\sqrt{x}}{dx} + \frac{d\sqrt{x}}{dx}\cdot x$$

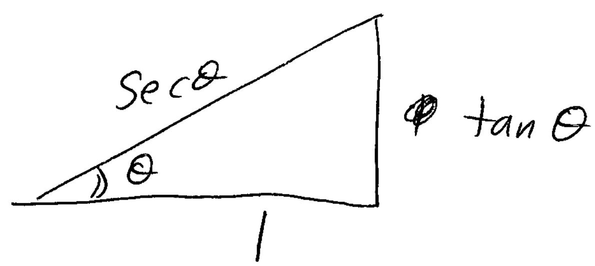
$$1 = 2\sqrt{x}\,\frac{d\sqrt{x}}{dx}$$

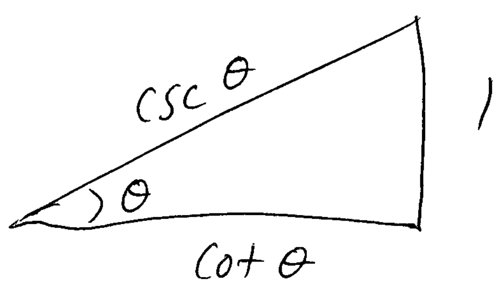
$$\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$$

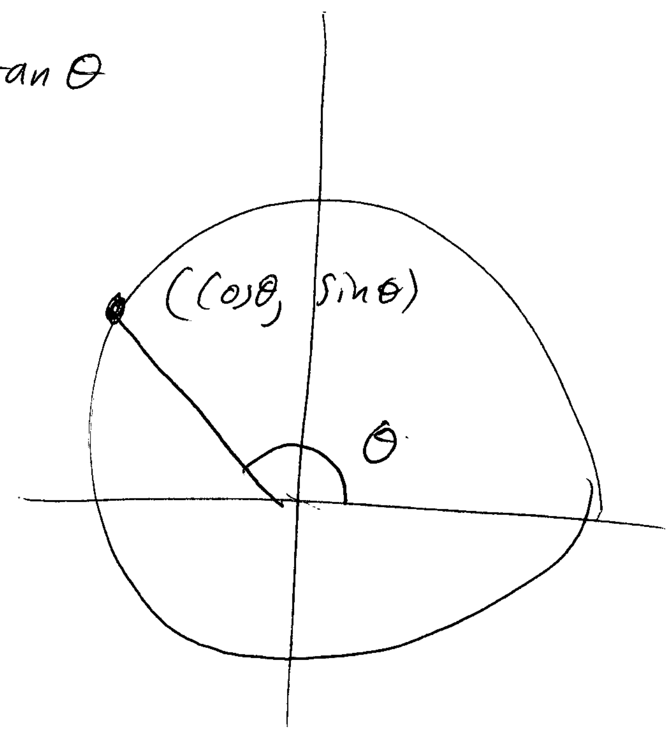
Trig.

Sohcahtoa!









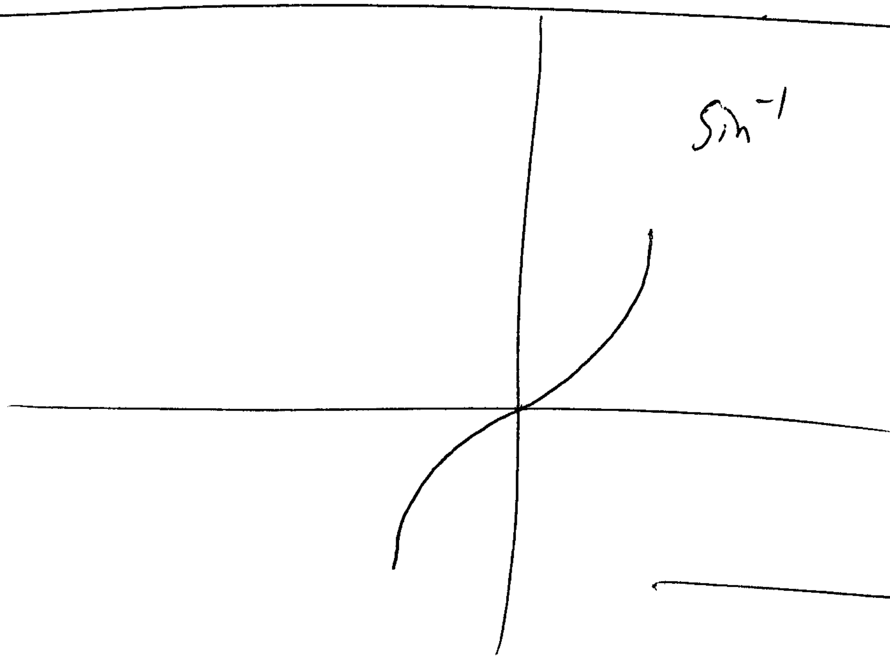
$$5^{\log_5 x} = x$$

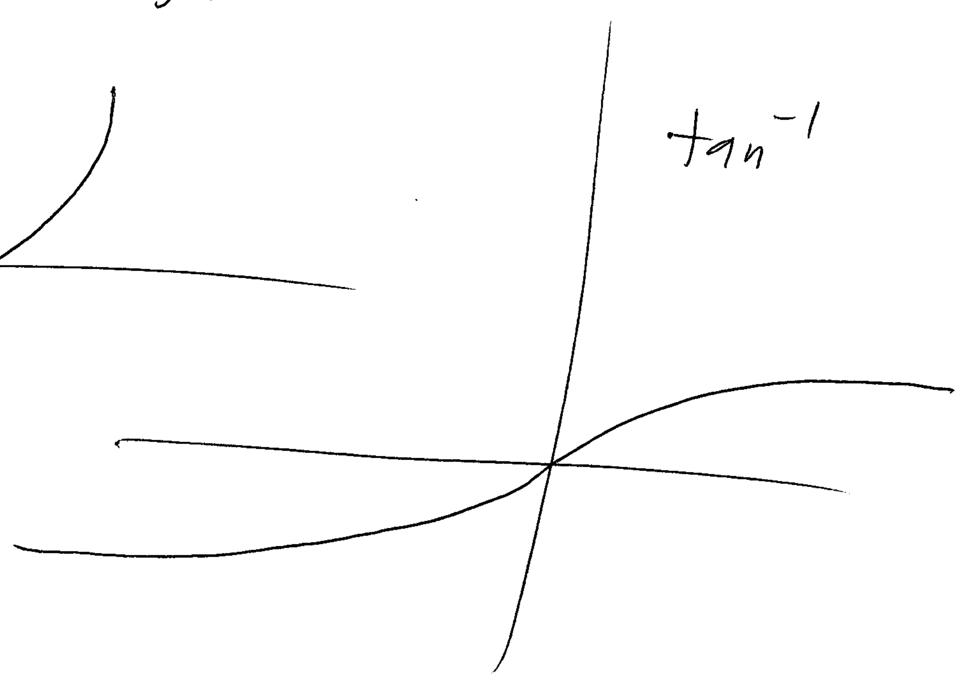
$$\log_5 (5^x) = x$$

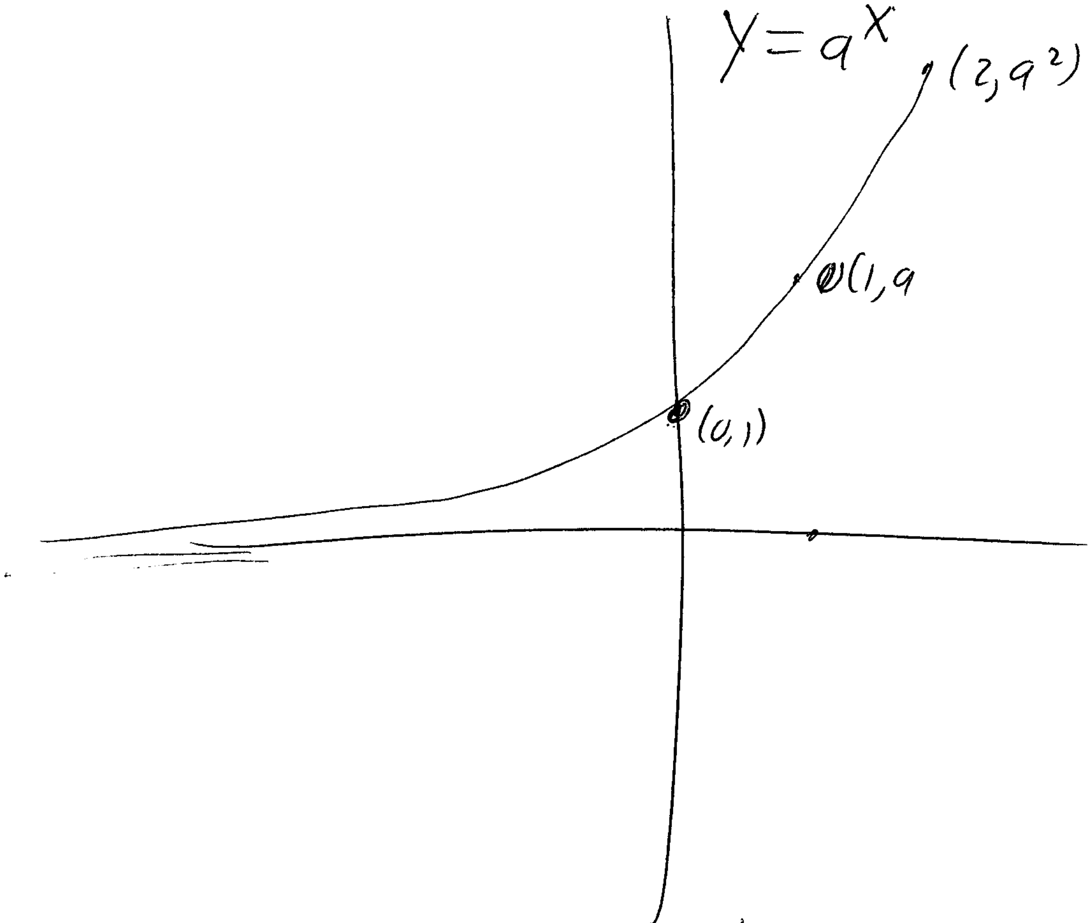
$$a^b a^c = a^{b+c}, \qquad (a^b)^c = a^{bc}$$

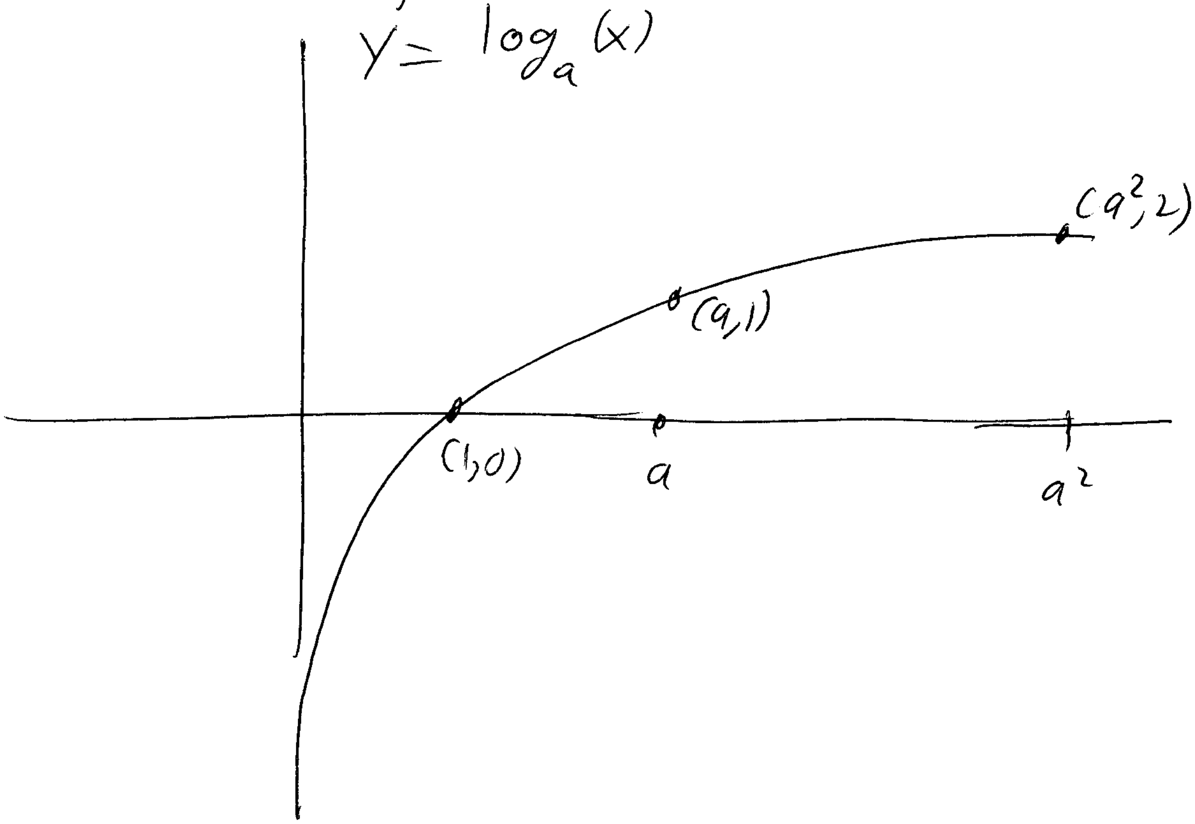
$$\log_a (xy) = \log_a(x) + \log_a(y)$$

$$\log_a (x^r) = r \log_a(x)$$






Y = a^x
(2, a^2)
(1, a
(0, 1)


Y = log_a (x)
(a^2, 2)
(a, 1)
(1, 0)
a
a^2

"$\lim_{x\to a} f(x) = L$" means "whenever $x \approx a$ (but not $= a$), $f(x) \approx L$"

$\lim_{x\to a^+} f(x) = L$ means "when x is slightly $> a$, $f(x) \approx L$"

$\lim (f \pm g) = \lim f \pm \lim g$

$\lim (fg) = (\lim f)(\lim g)$

$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$ if $\lim g \neq 0$.

If $f(x) = g(x)$ whenever $x \neq a$,

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$.

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} \left(\frac{(x-1)(x+1)}{x-1} \right)$$

$$= \lim_{x \to 1} (x+1) = 2$$

$$\sqrt{a} - \sqrt{b} = \frac{(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})}{(\sqrt{a} + \sqrt{b})} = \frac{a-b}{\sqrt{a} + \sqrt{b}}$$

Limits as $x \to \pm\infty \implies$ horizontal asymptotes

Infinite limits $\implies$ vertical "

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f is continuous at a if

$$\lim_{x \to a} f(x) = f(a)$$

Derivative = rate of change

= slope of tangent line

= Sensitivity.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$