M408N First Midterm Exam, September 27, 2011

1) Compute sec(sin⁻¹(3/5)). In other words, if sin(θ) = 3/5 and −π/2 < θ < π/2, what is sec(θ)?

2) If e³ln(x) = 8, what is x? Simplify your answer as much as possible.

3. Compute \( \lim_{x \to 3} \frac{x^2 - x - 6}{x - 3} \).

4. Let \( f(x) = \sqrt{2x^2 + 1} \), with a domain of \( x \geq 0 \). Find the formula for the inverse function \( f^{-1}(x) \).

5. Consider the function \( f(x) = \frac{x^2 - 1}{x^2 - 1} \). Find the vertical and horizontal asymptotes and sketch the graph \( y = f(x) \).

   Vertical asymptotes at:

   Horizontal asymptotes at:

   \[
   \begin{array}{c|c|c}
   \text{y} & 5 & 5 \\
   \hline
   -5 & \cdots & \cdots \\
   \hline
   -3 & \cdots & \cdots \\
   \hline
   5 & \cdots & \cdots \\
   \end{array}
   \]

6. Consider the function \( f(x) = \begin{cases} 
   x^2 & x > 3 \\
   3x & x \leq 3 
\end{cases} \). Is \( f(x) \) continuous? Why or why not?

7. Suppose the position of a particle at time \( t \) is given by the function \( f(t) = 2^{-t} \). (a) Graph position versus time on the first blank piece of graph paper. Be as precise as possible. (b) Sketch a graph of velocity versus time
on the second blank piece. This graph is not expected to be precise, but should be qualitatively right. You do not need the formula for the derivative of $2^{-t}$ to do this! Instead, I expect you to graph the derivative of $f(t)$ based on the shape of the graph of $f(t)$.
8. Consider the function \( f(x) = 5^x \). Which of the following expressions are equal to \( f'(2) \)? Circle all correct expressions — there may not be any, there may be one, or there may be more than one. For this problem (and only for this problem), explanations are unnecessary and will not be considered in the grading.

a) \( 25 \lim_{h \to 0} \frac{5^h - 1}{h} \)

b) \( 2(5)^{2-1} \)

c) \( \lim_{x \to 2} \frac{5^x - 25}{x - 5} \).

d) The slope of the line tangent to \( y = f(x) \) at \( (2, 25) \).

9. Let \( f(x) = \frac{1}{x} \). Compute \( f'(-4) \) FROM THE DEFINITION OF THE DERIVATIVE AS A LIMIT, making clear what you are doing at every step. (If you just plug into the formula for the derivative of \( x^n \) you will not get any credit.)

10. Suppose that \( f(3) = 4 \) and \( f'(3) = -1 \). Find the equation of the tangent line to \( y = f(x) \) at \( (3, 4) \).

For extra credit, use this tangent line to approximate \( f(3.05) \).