1. Let \( V = \mathbb{R}_2[t] \) with (standard) basis \( B = \{1, t, t^2\} \) and let \( W = M_{2,2} \) be the space of 2 by 2 real matrices with (standard) basis \( D = \{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \} \). Consider the linear transformation \( L(p) = \begin{pmatrix} p(1) - p(0) \\ p(2) - p(0) \\ p(-1) - p(0) \\ p(-2) - p(0) \end{pmatrix} \) from \( V \) to \( W \).

   a) Find the matrix of \( L \) relative to the bases \( B \) and \( D \).
   
   b) What is the dimension of \( \ker(L) \)? Find a basis for \( \ker(L) \).
   
   c) What is the dimension of \( \text{range}(L) \)? Find a basis for \( \text{range}(L) \).

2. Let \( A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 5 & 7 \\ 5 & 10 & 13 & 18 \end{pmatrix} \).

   a) Let \( V = \{ x \in \mathbb{R}^4 | Ax = 0 \} \). What is the dimension of \( V \)? Find a basis for \( V \).
   
   b) In \( \mathbb{R}^3 \), consider the vectors \( (1, 2, 5)^T, (2, 4, 10)^T, (3, 5, 13)^T \), and \( (4, 7, 18)^T \). Are these vectors linearly independent? Do they span \( \mathbb{R}^3 \)?
   
   c) Find a basis for the span of the four vectors of part (b).

3. Let \( L : \mathbb{R}^2 \to \mathbb{R}^2 \) be given by \( L \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8x_1 - 10x_2 \\ 3x_1 - 3x_2 \end{pmatrix} \). On \( \mathbb{R}^2 \), consider the standard basis \( \mathcal{E} \) and the alternate basis \( \mathcal{B} = \{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix} \} \). Finally, let \( \mathbf{v} = \begin{pmatrix} 8 \\ 3 \end{pmatrix} \).

   a) Find \( P_{\mathcal{EB}}, P_{\mathcal{BE}}, [\mathbf{v}]_{\mathcal{E}} \) and \( [\mathbf{v}]_{\mathcal{B}} \).
   
   b) Find the matrix \( [L]_{\mathcal{E}} \) and the matrix \( [L]_{\mathcal{B}} \).

4. The two parts of this problem are NOT connected.

   a) In \( \mathbb{R}_2[t] \), consider the vectors \( \mathbf{b}_1 = 1 + t + 2t^2, \mathbf{b}_2 = 2 + 3t + 5t^2 \) and \( \mathbf{b}_3 = 3 + 7t + 9t^2 \). Do these vectors form a basis for \( \mathbb{R}_2[t] \)? If so, find \( [\mathbf{v}]_{\mathcal{B}} \), where \( \mathbf{v} = 1 - 2t \). If not, find constants \( a_1, a_2, a_3 \), not all zero, such that \( a_1 \mathbf{b}_1 + a_2 \mathbf{b}_2 + a_3 \mathbf{b}_3 = 0 \).
   
   b) In \( \mathbb{R}_3[t] \), let \( V \) be the set of polynomials \( p \) for which \( p(0) = p(1) = 0 \). Find a basis for \( V \).
5. True or False? Each question is worth 4 points. You do NOT need to justify your answers, and partial credit will NOT be given.

a) The plane $x_1 + 3x_2 - 4x_3 = 0$ is a subspace of $\mathbb{R}^3$.

b) If $A$ is a $3 \times 5$ matrix, then the dimension of the null space of $A$ is at least 2.

c) Let $L : \mathbb{R}^5[t] \rightarrow \mathbb{R}^3$ be a linear transformation. If $L$ is onto, then the kernel of $L$ is 2-dimensional.

d) Let $\mathcal{B} = \{b_1, \ldots, b_n\}$ be a basis for a vector space $V$. If $n$ vectors $d_1, \ldots, d_n$ span $V$, then the vectors $[d_1]_B, \ldots, [d_n]_B$ are linearly independent.

e) Every linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^4$ is multiplication by a $5 \times 4$ matrix.