1. **Is it fixed yet?** Consider the system of nonlinear differential equations:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_1(3 - 2x_1 - x_2) \\
\frac{dx_2}{dt} &= x_2(5 - 2x_1 - 3x_2).
\end{align*}
\]

a) Find all the fixed points.

b) For each fixed point, indicate how many stable modes, and how many unstable modes, there are.

2. **Gram crackers.** In \( \mathbb{R}^3 \), with the standard inner product, consider the vectors \( \mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \), \( \mathbf{x}_2 = \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix} \), \( \mathbf{x}_3 = \begin{pmatrix} 16 \\ 9 \\ -2 \end{pmatrix} \) that form a basis for \( \mathbb{R}^3 \).

a) Use the Gram-Schmidt process to convert this basis to an orthogonal basis \( \{ \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3 \} \). (Note: the vectors \( \mathbf{y}_i \) do not have to be orthonormal, just orthogonal.)

b) The vector \( \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \) can be expressed as \( c_1 \mathbf{y}_1 + c_2 \mathbf{y}_2 + c_3 \mathbf{y}_3 \). Find \( c_1 \), \( c_2 \) and \( c_3 \).

3. **When least is best.** Find all least-squares solutions to the system of equations

\[
\begin{align*}
x_1 + 2x_2 &= 1 \\
2x_1 + 4x_2 &= 1 \\
x_1 + 6x_2 &= 1 \\
4x_1 + 8x_2 &= 1
\end{align*}
\]

4. **Working 24/7.** Consider the Hermitian matrix \( H = \begin{pmatrix} 24 & 7 \\ 7 & -24 \end{pmatrix} \).

a) Find the eigenvalues \( \lambda_1 \) and \( \lambda_2 \) and corresponding eigenvectors \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \) of \( H \).

b) Decompose \( \mathbf{x}_0 = \begin{pmatrix} 13 \\ 9 \end{pmatrix} \) as a linear combination of \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \).

c) If \( \frac{dx}{dt} = H \mathbf{x} \) and \( \mathbf{x}(0) = \mathbf{x}_0 \) as in (b), what is \( \mathbf{x}(t) \)?