1) Gram Schmidt:

a)(10 points) On $\mathbb{R}^3$ with the usual inner product, Use Gram-Schmidt to convert $\mathbf{x}_1 = (1, 2, 0)^T$, $\mathbf{x}_2 = (3, 1, 1)^T$, $\mathbf{x}_3 = (4, 3, -5)^T$ to an orthogonal basis.

b)(15 points) On $\mathbb{R}_2[t]$ with the inner product $\langle f | g \rangle = \int_0^2 f(t) g(t) dt$, transform $\{1, t, t^2\}$ to an orthogonal basis.

2. a)(15 points) Find the equation of the best line through the points $(1, -4)$, $(2, 1)$, and $(3, 2)$.

b)(10 points) Let $\mathbf{V}$ be the subspace of $\mathbb{R}^3$ that is the span of the vectors $(1, 2, 3)^T$ and $(1, 1, 1)^T$. Find the point in $\mathbf{V}$ that is closest to $(-4, 1, 2)^T$.

3. On $\mathbb{C}^3$ with the usual inner product, let

$$ L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + ix_2 - ix_3 \\ 2x_2 + (1 - i)x_3 \\ ix_1 + 3x_2 + x_3 \end{pmatrix} $$

a)(5 points) Find the matrix of $L$

b)(10 points) Let $\mathbf{x} = \begin{pmatrix} 1 \\ 10 \\ 100 \end{pmatrix}$. Compute $L^\dagger(\mathbf{x})$.

c)(10 points) Let $\mathbf{V}$ be the space of real-valued functions on the real line, with the inner product $\langle f | g \rangle = \int_{-\infty}^{\infty} f(t) g(t) dt$. Let $A : \mathbf{V} \rightarrow \mathbf{V}$ be the linear transformation $A(t) = t + d/dt$ (That is, $(A(f))(t) = tf(t) + f'(t))$. Let $g(t) = e^{-t^2/2}$. Compute $Ag$ and $A^\dagger g$.

4. Grab bag. These are short-answer or true/false questions. Each question is worth 5 points. You do NOT need to justify your answers, and partial credit will NOT be given.

a) True or false? The matrix $\begin{pmatrix} 5 & 4i \\ -4i & -1 \end{pmatrix}$ has orthogonal eigenvectors.

b) True or false? The matrix $\frac{1}{\sqrt{7}} \begin{pmatrix} 2 - i & -1 + i \\ 1 + i & 2 + i \end{pmatrix}$ is unitary.

c) Let $\mathbf{x}(t)$ be the solution to $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, where $A = \begin{pmatrix} 0 & 1 & 2 & 3 \\ -1 & 0 & -1 & 4 \\ -2 & 1 & 0 & 5 \\ -3 & -4 & -5 & 0 \end{pmatrix}$ and $\mathbf{x}(0) = (5, -3, 1, 1)^T$. Find the limit, as $t \rightarrow \infty$, of $|\mathbf{x}(t)|$. (This has a quick
and easy solution, and you do NOT have to diagonalize $A$!

d) True or false? If a matrix $M$ satisfies $M = M^T$, then the eigenvalues of $M$ are real.

e) True or false? If a matrix is unitary, then it is not Hermitian.