M408M First Midterm Exam, September 26, 2013

1) (28 pts, 2 pages) Consider the parametrized curve in the plane given by

$$\begin{aligned} x &= 10\cos(t) + 8 \\ y &= 6\sin(t), \end{aligned}$$

where t runs from 0 to 2π. This traces out an ellipse.

a) Find the equation of this ellipse in rectangular coordinates.

b) Find the slope of the tangent to the ellipse at the point $(13, 3\sqrt{3})$ (that is, when $t = \pi/3$).

c) Find the location of the foci of the ellipse. What is the eccentricity?

d) Find an equation for the ellipse in polar coordinates. Express your answer in the form $r =$ (some function of θ).

2. (24 points) Consider the polar curves $r = \sqrt{3 + 2\sin(\theta)}$ and $r = 2$.

a) Where do these two curves intersect? (You can express your answer either in polar or rectangular coordinates. It's your choice, but be clear about which you are using.)

b) Find the area of the region inside the first curve ($r = \sqrt{3 + 2\sin(\theta)}$) and outside the second ($r = 2$).

c) What is the perimeter of the region whose area you computed in (b)? The answer involves a **nasty** definite integral that you are **NOT** expected to evaluate! Just leave that part of the answer as an integral. (That is, your answer might be something like $4\pi + \int_{-3}^{2} e^{-\theta^2} d\theta$, although that isn't actually the answer.)

3. (48 pts, 2 pages) In 3-dimensional space, consider a triangle with vertices at P(1,2,3), Q(7,10,3) and R(4,6,-2).

a) Find the length of all three sides of the triangle.

b) Compute the vectors $\vec{u} = \overrightarrow{PQ}$ and $\vec{v} = \overrightarrow{PR}$, and then compute the dot product $\vec{u} \cdot \vec{v}$.

c) Find the angle at P formed by the legs of the triangle.

d) Compute the component $Comp_{\vec{u}}\vec{v}$ and projection $Proj_{\vec{u}}\vec{v}$ of $\vec{v}$ in the $\vec{u}$ direction.

d) Compute the cross product $\vec{u} \times \vec{v}$.

e) Find the area of the triangle.